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Complex crystallographic groups I

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§ 0. Introduction.

Let $\boldsymbol{E}(n)$ be the complex motion group acting on the n-dimensional complex euclidean space $X=\boldsymbol{C}^n$. We define a crystallographic group Γ on X to be a discrete subgroup of $\boldsymbol{E}(n)$ with compact quotient. The problem is to find crystallographic groups and to investigate the structure of the quotient space $M=X/\Gamma$ and the ramification of the natural map $X\rightarrow M$.

For $n=1$, solution of the problem is well known. For $n\geqq 2$, several authors studied fixed point free crystallographic groups and the quotient manifolds by them ([**14**], [**15**], [**17**]). We are interested in the groups which admit fixed points. For two dimensional crystallographic reflection groups, Shvartsman ([**13**]) proved the rationality of the quotient spaces and determined all splittable groups and the quotient spaces.

In this paper, we study geometric and topological properties of the quotient varieties and prove the vanishing of the plurigenera under certain conditions and obtain the characterization of crystallographic reflection groups. Two dimensional groups whose point groups are generated by reflections are investigated in detail: We prove that the quotient varieties are rational except for the group $\Gamma_{NR}(\tau)$ and the desingularization of the quotient variety by $\Gamma_{NR}(\tau)$ is an Enriques surface. All the two dimensional crystallographic reflection groups and the quotient varieties by them are determined.

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§ 1. Preliminaries and notations.

1.1. We shall use the following notations:

$X\cong\boldsymbol{C}^n$: n-dimensional complex euclidean space

$\boldsymbol{A}(n)=\{(A, a)\mid A\in GL(n, \boldsymbol{C}),\ a\in\boldsymbol{C}^n\}$: affine transformation group on X