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An axiomatization theorem

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Let L be a first order finitary predicate logic with equality $L_{\omega,\omega}$, or a first order infinitary predicate logic with equality $L_{\omega_1,\omega}$, and I, J, K three sets of formulas in L. Then, I and J are said to be equivalent in L over K if A is provable from I in L iff A is provable from J in L, for any formula A in K. Also, I is said to be an axiomatization of J in L, if I is a subset of J and, I and J are equivalent in L over J. An axiomatization theorem of J in L is a statement to give us a "concrete" method to construct a "simple" axiomatization of J in L. Of course, "concrete" and "simple" have no precise mathematical meanings and we use them rather informally. But, two remarks on them will be given in the following. First, if $L=L_{\omega,\omega}$ and J is a recursively enumerable set of formulas in L under a nice Gödel-numbering, then there is a method to construct a primitive recursive axiomatization of J in L by the well-known theorem due to W. Craig (cf. Craig [2]). But the axiomatization of J obtained by Craig's method seems to be so complicated that, in practice, one can not easily tell whether or not a given formula belongs to it, generally (cf. p. 141 in Keisler [4]). This shows us that it is meaningful to give a concrete method to construct a simple axiomatization of J, even if J is recursively enumerable. Secondly, the set J is usually defined using some parameters $p_1, p_2, \cdots, p_n$. So, in order to give an axiomatization theorem of J in L, we should clearly state how to construct an axiomatization of J from each values of parameters $p_1, p_2, \cdots, p_n$, concretely. To define sets of formulas which we are going to deal with, and to state their axiomatization theorem, we require some definitions. Suppose that W is a set of predicate symbols. Then, W-free (W-positive, W-negative) formulas are formulas which have no (no negative, no positive) occurrences of predicate symbols in W. W-atomic formulas are formulas of the form $P(\bar{t})$ for some $P\in W$ and some sequence $\bar{t}$ of terms. A formula A is said to belong to a formula B syntactically, if every predicate symbol occurring in A positively (negatively) occurs in B positively (negatively). For each sentence A in L, and each two sets S, Q of predicate symbols in L, let

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