J. Math. Soc. Japan
Vol. 34, No. 3, 1982

On the topology of the Newton boundary III

By Mutsuo OKA

(Received Oct. 8, 1980)
(Revised Feb. 23, 1981)

§ 1. Introduction.

The purpose of this paper is to prove the following theorem: the Milnor fibration of an analytic function $f(z)$ is uniquely determined by the Newton boundary $\Gamma(f)$ if f is non-degenerate. We have proved the assertion in [**4**] for the case that the origin is an isolated critical point of f and in [**5**] for a weighted homogeneous polynomial. However the proof for the general case involves several essential arguments. For instance, we shall show in the process of the proof that the stable radius of the Milnor fibration of f is obtained by the Newton boundary $\Gamma(f)$. (Theorem 1, § 1).

We use the following notations. The other notations and terminology are the same as in [**4**] and [**5**].

$$S_r=\{z\in \boldsymbol{C}^n;\ \|z\|=r\}\,,\quad B_r=\{z\in \boldsymbol{C}^n;\ \|z\|\leqq r\}\,,\quad \mathrm{Int}(B_r)=\{z\in \boldsymbol{C}^n;\ \|z\|<r\}$$

and $S_r^1=\{u\in \boldsymbol{C};\ |u|=r\}$.

§ 2. A stable radius.

Let $f(z)$ be an analytic function of n variables which is defined in the neighborhood of the origin and assume that $f(\vec{0})=0$. Recall that a stable radius of the Milnor fibration of f is a positive number ε which satisfies the following condition.

(T) For any r, $0<r\leqq\varepsilon$, there exists a positive number $d(r)$ such that for any non-zero u, $|u|\leqq d(r)$, the hypersurface $f^{-1}(u)$ is non-singular in B_ε and it meets transversely with the spheres S_h for any h, $r\leqq h\leqq\varepsilon$.

The existence of a stable radius ε is proved by Hamm-Lê. (Lemme (2.1.4) of [**1**]). For any $0<r\leqq\varepsilon$, and $0<d\leqq d(r)$, let $E(r, d)$ be $f^{-1}(S_d^1)\cap\mathrm{Int}(B_r)$. The restriction of f to $E(r, d)$ gives a locally trivial fibration over S_d^1, say $\xi(r, d; f)$ and the isomorphism class of $\xi(r, d; f)$ does not depend on the particular choice

This research was partially supported by Grant-in-Aid for Scientific Research (No. 56740014), Ministry of Education.