J. Math. Soc. Japan
Vol. 34, No. 3, 1982

On meromorphic maps into a compact complex manifold

By Hirotaka FUJIMOTO

(Received Feb. 10, 1981)

§ 1. Introduction.

In [8], the author has shown that, for any given hyperplanes $H_1, \cdots, H_{N+2}$ in $P^N(\boldsymbol{C})$ located in general position and effective divisors $E_1, \cdots, E_{N+2}$ on $\boldsymbol{C}^n$, the set $\mathcal{F} := \mathcal{F}\binom{H_1, \cdots, H_{N+2}}{E_1, \cdots, E_{N+2}}$ of all non-degenerate meromorphic maps of $\boldsymbol{C}^n$ into $P^N(\boldsymbol{C})$ such that the pull-backs $f^*(H_i)$ $(1 \leqq i \leqq N+2)$ of divisors H_i are equal to E_i respectively is finite. The purpose of this paper is partly to prove that the number of maps in the above set $\mathcal{F}$ is bounded by a constant depending only on N and mainly to generalize this result to the case of meromorphic maps into a compact complex manifold.

Let M be an N-dimensional connected compact complex manifold and L be a line bundle over M. We denote by $H^0(M, \mathcal{O}(L))$ the set of all holomorphic sections of L and by (ϕ) the divisor of zeros of a non-zero section $\phi \in H^0(M, \mathcal{O}(L))$. Set

$$|L| = \{(\phi);\ \phi \in H^0(M, \mathcal{O}(L)),\ \phi \not\equiv 0\}.$$

DEFINITION 1.1. Let $D_1, \cdots, D_m \in |L|$ such that $D_i = (\phi_i)$ $(1 \leqq i \leqq m)$ for $\phi_i \in H^0(M, \mathcal{O}(L))$. We define $\phi_1, \cdots, \phi_m$ (or $D_1, \cdots, D_m$) to be algebraically independent if there exists no non-zero homogeneous polynomial $P(w_1, \cdots, w_m)$ satisfying the relation

$$P(\phi_1, \cdots, \phi_m) \equiv 0$$

in $H^0(M, \mathcal{O}(L^d))$, where $d = \deg P$.

DEFINITION 1.2. A meromorphic map $f: \boldsymbol{C}^n \to M$ is said to be *algebraically non-degenerate* with respect to L if there exists no non-zero holomorphic section $\phi \in H^0(M, \mathcal{O}(L^d))$ $(d > 0)$ such that $f(\boldsymbol{C}^n) \subseteqq \{\phi = 0\}$.

Take $N+2$ divisors $D_1, \cdots, D_{N+2} \in |L|$ and effective divisors $E_1, \cdots, E_{N+2}$ on $\boldsymbol{C}^n$. Let $\mathcal{F}\binom{D_1, \cdots, D_{N+2}}{E_1, \cdots, E_{N+2}}$ denote the set of all meromorphic maps of $\boldsymbol{C}^n$ into M which are algebraically non-degenerate with respect to L such that the

This research was partially supported by Grant-in-Aids for Scientific Research (No. 564049 and No. 546004), Ministry of Education.