J. Math. Soc. Japan
Vol. 34, No. 3, 1982

Weak L-spaces are free L-spaces

By Munehiko ITŌ

(Received Sept. 6, 1980)
(Revised Jan. 23, 1981)

1. Introduction.

In order to discuss the dimension theory, K. Nagami [4], [5] introduced the concepts of free L-spaces and weak L-spaces. He posed the following two problems in [5] and [4] respectively.

1. Does the class of weak L-spaces coincide with the class of free L-spaces ?
2. Is the perfect image of a free L-space again a free L-space ? (Problem 2.11.)

The main purpose of this paper gives a positive answer to the first problem. In Section 4 we give a partial answer to the second problem as follows.

The closed continuous image of a free L-space need not be a free L-space.

In this paper all spaces are assumed to be Hausdorff topological spaces. The letter N denotes the positive integers. For undefined terminology refer to [2].

The author thanks Professor K. Nagami for his guidance.

2. Definition.

DEFINITION 2-1. Let X be a space and F a closed subset of X. A family $\mathcal{U}$ of open sets is said to be an *anti-cover* of F if $\mathcal{U}^{\#}(=\bigcup\{U: U\in\mathcal{U}\})=X-F$.

Let $\mathcal{U}$ be an anti-cover of F. For a subset S of X $\mathrm{St}^i_{\mathcal{U}}(S)$ is defined inductively by the formulae

$$\mathrm{St}^1_{\mathcal{U}}(S)=\mathrm{St}_{\mathcal{U}}(S)=\{U\in\mathcal{U}: U\cap S\neq\emptyset\}^{\#},$$

$$\mathrm{St}^i_{\mathcal{U}}(S)=\mathrm{St}_{\mathcal{U}}(\mathrm{St}^{i-1}_{\mathcal{U}}(S)).$$

An open neighborhood W of F is said to be a *canonical* (*semi-canonical*) *neighborhood* of F with respect to $\mathcal{U}$ if $F\cap\mathrm{Cl}\,\mathrm{St}^i_{\mathcal{U}}(X-W)=\emptyset$ for each $i\in N$ ($F\cap\mathrm{Cl}\,\mathrm{St}_{\mathcal{U}}(X-W)=\emptyset$) respectively.

Let $\mathcal{W}=\{W_a: a\in A\}$ be a family of neighborhoods of F. $\mathcal{W}$ is said to be an *anti-closure-preserving family* if $\{(X-W_a)\cup F: a\in A\}$ is closure-preserving.

DEFINITION 2-2. For a space X consider a pair $\mathcal{P}=(\mathcal{F}, \{\mathcal{U}_F: F\in\mathcal{F}\})$ such that $\mathcal{F}$ is a family of closed sets of X and each $\mathcal{U}_F$ is an anti-cover of F. $\mathcal{P}$