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Note on γ-dimension and products of real projective spaces

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1. Introduction.

Let α be the stable class of a vector bundle over a complex X. The γ-dimension, $\dim_\gamma \alpha$, of α is defined as follows (cf. [**6**]):

$$\dim_\gamma \alpha = \sup\{i \in N \mid \gamma^i(\alpha) \neq 0\},$$

where N is the set of positive integers and γ^i is the i-th Grothendieck γ-operation (cf. [**2**]). Let $\tau_0(M)$ denote the stable class of the tangent bundle $\tau(M)$ of a differentiable manifold M. H. Suzuki [**5**] investigated $\dim_\gamma \tau_0(P^m \times P^n)$ and $\dim_\gamma(-\tau_0(P^m \times P^n))$, where P^n is the n-dimensional real projective space, and applied them to the problem of vector fields on $P^m \times P^n$ and to the problem of immersions and embeddings of $P^m \times P^n$ in Euclidean spaces. The purpose of this note is to improve Suzuki's results.

Let $\varphi(n)$ be the number of integers s such that $0 < s \leqq n$ and $s \equiv 0$, 1, 2 or 4 mod 8, $[a]$ be the integral part of a real number a, and $\binom{k}{i}$ be a binomial coefficient $k!/(k-i)!\,i!$. Define integers $\delta(n)$ and $\delta(m, n)$ as follows:

$$\delta(n) = \max\left\{i > 0 \;\middle|\; 2^{i-1}\binom{n+1}{i} \not\equiv 0 \bmod 2^{\varphi(n)}\right\},$$

$$\delta(m, n) = \max\left\{i > 1 \;\middle|\; 2^{i-2}\left\{\binom{m+n+2}{i} - \binom{m+1}{i} - \binom{n+1}{i}\right\} \not\equiv 0 \bmod 2^{[l/2]}\right\},$$

where $l = \min\{m, n\}$. Then we prove

THEOREM 1. $\dim_\gamma \tau_0(P^m \times P^n) \geqq \delta(m, n)$.

If $m = n = 2^r - 2$ $(r \geqq 3)$, then $\delta(m, n) = 2^{r-1} = \delta(n) + 1 > \delta(n)$. Therefore Theorem 1 is a partial improvement of [**5**, (4.2)]. But if $m = 2^r - 2$ and $n \leqq 2^{r-1} - 2$ $(r \geqq 3)$ then $\delta(m, n) \leqq 2^{r-2} < 2^{r-1} - 1 = \delta(m)$ and hence in this case [**5**, (4.2)] is better than the theorem. Combining [**5**, (4.2)] and Theorem 1, we obtain

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