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Exceptional values for meromorphic solutions of some difference equations

By Niro YANAGIHARA

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1. Introduction.

In this note, we consider the non-linear difference equation

(1.1) $$y(x+1)=R(y(x)),$$

where $R(x)$ is a rational function with the degree p, $p\geqq 2$.

Julia [**1**, p. 158] proved that

either there is a number λ such that

(1.2) $$\lambda=R(\lambda), \qquad R'(\lambda)=1,$$

or there is a number λ such that

(1.3) $$\lambda=R(\lambda), \qquad |R'(\lambda)|>1.$$

In either case, the equation (1.1) has a meromorphic solution determined as follows.

Let λ be a number for which (1.2) holds. Putting

(1.2-1) $$y(x)=\lambda+1/w(x),$$

we obtain

(1.2-2) $$w(x+1)=w(x)\left[1-\frac{R^{(m+1)}(\lambda)}{(m+1)!}w(x)^{-m}+\cdots\right] \qquad (m\geqq 1)$$

$$=R_1(w(x)), \quad \text{with a rational function } R_1(x).$$

Further, if we put

(1.2-3) $$\omega(x)=w(x)^m/A^m, \qquad A=\left[\frac{-m}{(m+1)!}R^{(m+1)}(\lambda)\right]^{1/m},$$

then we get

(1.2-4) $$\omega(x+1)=F(\omega(x)),$$

where

(1.2-5) $$F(x)=x+1+\sum_{j\geq m+1} b_j x^{1-j/m}.$$

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