J. Math. Soc. Japan
Vol. 34, No. 3, 1982

Good reduction of elliptic modules

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(Received Jan. 6, 1981)

In this paper we give a criterion for good reduction of elliptic modules (Theorem 1, Section 2) which is an analogue of the criterion of Néron-Ogg-Šafarevič for abelian varieties, cf. [7]. In the rest of the paper we give applications to elliptic modules of rank one over global function fields: In Section 3, the main theorem of complex multiplication of elliptic modules ([3] and [5]) is reformulated in a more relevant form to our subject (Theorem 2). Then, to each elliptic module we can associate the "Hecke character" (Theorem 3) so that the elliptic module has good reduction at a place v if and only if the Hecke character is unramified at v. In Section 4, we give a classification theorem (Theorem 4) by means of the Hecke characters. As an application, it will be shown that each rank-one elliptic module over a global function field K has a K-form which has good reduction everywhere (Theorem 5).

1. Elliptic modules.

In this section we recall briefly the basic concepts of elliptic modules. For details, see [3] and [5].

Let F be a global field of characteristic $p>0$, $\mathbf{F}_q$ the finite field of constants, ∞ a fixed prime divisor and A the ring of elements of F which are integral outside ∞. For a commutative ring K of characteristic p we let denote $K\{\phi\}$ the (non commutative) ring of polynomials in ϕ over K with the relation $\phi c = c^q\phi$ for $c\in K$. When K is an A-algebra, i.e., there is defined $i: A\to K$, the ideal $\operatorname{Ker} i$ of A is called the *divisorial characteristic* of K (notation: $\operatorname{div\,char} K$). An *elliptic A-module X over* an algebra K is a ring homomorphism $f: A\to K\{\phi\}$ satisfying the following three conditions:

(a) $D\circ f=i$, where $D: K\{\phi\}\to K$ is a homomorphism defined by $D(\sum c_j\phi^j)=c_0$.

(b) The leading coefficient of $f(a)$ is invertible in K for each nonzero element a of A.

(c) The image $f(A)$ is not contained in K.

We write $[a]_X$, or simply a_X, for the image $f(a)$ of $a\in A$ under f. If $a_X=$

This research was partially supported by Grant-in-Aid for Scientific Research (No. 554020), Ministry of Education.