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The role of boundary Harnack principle in the study of Picard principle

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A nonnegative locally Hölder continuous function P on $0<|z|\leqq 1$ will be referred to as a *density* on $\Omega: 0<|z|<1$. Here we consider Ω as an end of the punctured sphere $0<|z|\leqq+\infty$ so that the point $z=0$ is viewed as the ideal boundary $\delta\Omega$ of Ω, the unit circle $|z|=1$ as the relative boundary $\partial\Omega$ of Ω, and the punctured closed unit disk $0<|z|\leqq 1$ as the relative closure $\bar{\Omega}$ of Ω. Similar notations are used for subregions of Ω. For example we denote by $\partial\Omega_a$ and $\bar{\Omega}_a$ the relative boundary $|z|=a$ and the relative closure $0<|z|\leqq a$ of the subregion $\Omega_a: 0<|z|<a$ $(a\in(0, 1])$ of Ω, respectively. A density P on Ω gives rise to an elliptic operator $L=L_P$ on Ω defined by

$$Lu=L_P u=\Delta u-Pu\,, \qquad \Delta=\partial^2/\partial x^2+\partial^2/\partial y^2\,.$$

Since $\delta\Omega$ is of parabolic character, there exists a unique bounded solution $e(z, a)=e_P(z, a)$, referred to as the *P-unit* on Ω_a, of $Lu=0$ on Ω_a with continuous boundary values 1 on $\partial\Omega_a$. We simply denote by $e(z)=e_P(z)$ the P-unit $e(z, 1)=e_P(z, 1)$ on Ω. With the operator $L=L_P$ we associate an elliptic operator $\hat{L}=\hat{L}_P$ on Ω, referred to as the *associate operator* to $L=L_P$, given by

$$\hat{L}v=\hat{L}_P v=\Delta v+2\nabla\log e_P\cdot\nabla v\,, \qquad \nabla=(\partial/\partial x, \partial/\partial y)\,.$$

After Bouligand we say that the *Picard principle* is valid for P at $\delta\Omega$ if the dimension of the half module of nonnegative solutions of $Lu=0$ on Ω with continuous boundary values 0 on $\partial\Omega$ is 1. We also say that the *Riemann theorem* is valid for $\hat{L}$ at $\delta\Omega$ if the limit $\lim_{z\to\delta\Omega}v(z)$ exists for every bounded solution v of $\hat{L}v=0$ on Ω. Then we have the following *duality theorem* (cf. Heins [3], Hayashi [2], Nakai [8]): The Picard principle is valid for P at $\delta\Omega$ if and only if the Riemann theorem is valid for $\hat{L}$ at $\delta\Omega$. As a sufficient condition of the Riemann theorem for $\hat{L}$ at $\delta\Omega$ we have, what we call, the following *boundary Harnack principle* for L at $\delta\Omega$:

(1) *For every a in* $(0, 1]$ *there exists a Jordan curve K_a in Ω_a which separates $\delta\Omega$ from $\partial\Omega_a$ and satisfies* $C(K_a; \Omega_a, L)=O(1)$ $(a\to 0)$, *where for every*

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