J. Math. Soc. Japan
Vol. 34, No. 3, 1982

Equivariant embeddings and isotopies of a sphere in a representation

Dedicated to Professor Kentaro Murata on his 60 th birthday

By Katsuhiro KOMIYA

(Received Nov. 11, 1980)

§1. Introduction and statement of results.

Let G be a finite group. Let M, N be smooth (i.e., infinitely differentiable) G-manifolds, and R the real line with trivial G-action. A level preserving smooth G-embedding

$$H: M\times R \longrightarrow N\times R$$

defines smooth G-embeddings H_t of M in N, for all $t\in R$, by the relation

$$H(x, t)=(H_t(x), t) \qquad \text{for any} \quad x\in M.$$

Let $f, g: M\rightarrow N$ be smooth G-embeddings. If, for some $a<b$,

$$H_t=f \qquad \text{for any} \quad t\leqq a,$$

$$H_t=g \qquad \text{for any} \quad t\geqq b,$$

then H is called a smooth G-isotopy between f and g, and f, g are called to be G-isotopic. The isotopy class $[f]$ is the set of all smooth G-embeddings which are G-isotopic to f. Denote by $\mathrm{Iso}^G(M, N)$ the set of all isotopy classes of smooth G-embeddings of M in N.

Let U be a finite dimensional representation of G. $S(U)$ denotes the unit sphere in U with respect to some G-invariant inner product. Then $S(U)$ is a smooth G-manifold. The purpose of this paper is to enumerate $\mathrm{Iso}^G(S(U), V)$ for finite dimensional representations U, V of G. In this paper we restrict ourselves to the case

$$\text{(C)} \qquad 0<\dim U^G<\dim U,$$

where U^G is the fixed point set in U by the G-action. All representations considered are real representations, and dim denotes the real dimension.

This research was partially supported by Grant-in-Aid for Scientific Research (No. 474028), Ministry of Education.