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Linear evolution equations $du/dt+A(t)u=0$: a case where $A(t)$ is strongly uniform-measurable

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§ 1. Introduction.

Kato [1, 2] studied the Cauchy problem for a linear evolution equation of hyperbolic type in a Banach space X:

$$(d/dt)u(t)+A(t)u(t)=0\,, \quad u(s)=y\in Y\,, \quad 0\leqq s\leqq t\leqq T<\infty\,,$$

where Y is a Banach space dense in X and $-A(t)$ is the generator of a (C_0) semigroup of bounded linear operators on X for each t. He proved a basic existence theorem (Theorem 4.1 of [1]) of the solution for the Cauchy problem when the family $A=\{A(t)\}$ is stable (see P. 244 of [1]) and $A(\cdot)$ is (Y, X) norm-continuous, i.e., $A(t)$ belongs to $\boldsymbol{B}(Y, X)$ and it is continuous in the norm of $\boldsymbol{B}(Y, X)$. Here $\boldsymbol{B}(Y, X)$ denotes the set of all bounded linear operators on Y to X. Though he used Cauchy's method in the proof, the author gave another proof by means of the Yosida approximation in [6]. Kato also proposed to solve the Cauchy problem when $A(\cdot)$ is (Y, X) strongly continuous.

In this paper we prove an existence theorem (Theorem 2.1) directly by the Yosida approximation method for a case where $A(\cdot)$ is (Y, X) strongly uniform-measurable. Since our method involves no process of step function approximations of time-dependent operators, it is distinguished from Cauchy's method as well as from the usual Yosida approximation method for evolution equations (see [7, 8]). Some additional assumption ((A4) (c) in § 2) is needed for the proof but we hope it is not so restrictive. We remark that Kobayasi [9] obtained a similar result by Cauchy's method with no additional assumptions when $A(\cdot)$ is (Y, X) strongly continuous but it seems difficult to extend his result to a case where $A(\cdot)$ is (Y, X) strongly measurable.

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§ 2. Theorem.

In this section we state our theorem with some preliminary considerations. Our assumptions are the following.