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Integral representation of an analytic functional

By Yoshimichi TSUNO

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1. Introduction.

An analytic functional is a continuous linear functional on the space of all holomorphic functions in some set in the complex n dimensional space $\boldsymbol{C}^n$. For an open set U in $\boldsymbol{C}^n$, we denote by $\mathcal{O}(U)$ the space of all holomorphic functions in U equipped with the compact convergence topology. It is a Fréchet space. When K is a compact set in $\boldsymbol{C}^n$, $\mathcal{O}(K)$ is the space of all functions holomorphic in some open neighborhood U of K equipped with the inductive limit topology of $\mathcal{O}(U)$ for all such U. It is a DF space and its topological dual space $\mathcal{O}'(K)$ is a Fréchet space. When $n=1$, $\mathcal{O}'(K)$ is determined by S. e. Silva, G. Köthe and A. Grothendieck. It is known as the following isomorphism:

$$\mathcal{O}'(K)\cong\mathcal{O}(V-K)/\mathcal{O}(V),$$

where V is an open neighborhood of K. The duality is explicitly given by

$$\langle f, g\rangle=\int_{\partial U} f(z)g(z)dz$$

where $f\in\mathcal{O}(K)$, $g\in\mathcal{O}(V-K)$ and U $(K\subset U\Subset V)$ is taken so that $f\in\mathcal{O}(\bar{U})$ and ∂U is smooth. This duality formula is independent of the choice of the open set U and the function g in the class $[g]$ in $\mathcal{O}(V-K)/\mathcal{O}(V)$. When $n>1$, this isomorphism is extended by A. Martineau and R. Harvey (cf. H. Komatsu [6]) as the form

$$\mathcal{O}'(K)\cong H^{n-1}(V-K, \mathcal{O})$$

under the conditions $H^j(K, \mathcal{O})=0$ $(j\geqq 1)$ where $\mathcal{O}$ is the sheaf of germs of holomorphic functions and V is a Stein neighborhood of K. The proof of this duality depends on the Serre duality theorem and is given by the functional analytic method. The purpose of this paper is to give a new proof of this duality theorem establishing the direct duality formula between these two spaces $\mathcal{O}(K)$ and $H^{n-1}(V-K, \mathcal{O})$. We will interpret the cohomology space $H^{n-1}(V-K, \mathcal{O})$ as the Dolbeault cohomology space and establish the duality through the formula:

$$\langle f, g\rangle=\int_{\partial U} f(z)g(z)\wedge dz_1\wedge dz_2\wedge\cdots\wedge dz_n,$$