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On the boundary behavior of superharmonic functions in a half space

By Yoshihiro MIZUTA

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1. Introduction.

A non-negative superharmonic function u in the half space $D=\{x=(x_1, \cdots, x_n) \in R^n ; x_n>0\}$, $n\geqq 2$, is represented as

$$u(x)=ax_n+\int_D G(x, y)d\mu(y)+\int_{\partial D} P(x, y)d\nu(y), \qquad x\in D,$$

where a is a non-negative number, μ (resp. ν) is a non-negative measure on D (resp. ∂D), G is the Green function for D and P is the Poisson kernel for D. It is known in [4] that

$$\lim_{x\to O,\, x\in D-E} x_n^{-1}u(x)=a+b_n\int \frac{y_n}{|y|^n}d\mu(y)+c_n\int \frac{1}{|y|^n}d\nu(y),$$

$$\lim_{x\to O,\, x\in D-E} x_n^{-1}|x|^n\{u(x)-ax_n\}=c_n\nu(\{O\})$$

for a Borel set $E\subset D$ which is minimally thin at O, where

$$b_n=\begin{cases} 2(n-2) & \text{if } n\geqq 3, \\ 2 & \text{if } n=2, \end{cases} \qquad c_n=\pi^{-n/2}\Gamma(n/2).$$

Our aim in this note is to show that $x_n^{-\beta}|x|^{\beta+\gamma}\{u(x)-ax_n\}$, $0\leqq\beta\leqq 1$, $-1\leqq\gamma\leqq n-1$, has a limit as $x\to O$ with an exceptional set, for which we shall give a metrical estimate of Wiener type. To do this, we shall study the boundary behavior of the Green potential $G_\alpha(x, \mu)=\int_D G_\alpha(x, y)d\mu(y)$, where

$$G_\alpha(x, y)=\begin{cases} |x-y|^{\alpha-n}-|\bar{x}-y|^{\alpha-n} & \text{in case } 0<\alpha<n, \\ \log(|\bar{x}-y|/|x-y|) & \text{in case } \alpha=n=2, \end{cases}$$

$\bar{x}$ denoting the reflection of x with respect to the surface ∂D, i.e.,

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