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Elimination of certain Thom-Boardman singularities of order two

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§ 0. Introduction.

In this paper we will study the problem of deforming a differentiable map of a differentiable manifold N into a differentiable manifold P in the homotopy class to a differentiable map which does not admit particularly complicated Thom-Boardman singularities of order two.

Let $\Sigma^I(N, P)$ denote the Thom-Boardman singularity with symbol I which is defined in the jet-space $J^r(N, P)$ where I denote either (i) for $r=1$, or (i, j) for $r=2$ ([2], [11] and [18]). Let $\bar{\Sigma}^I(N, P)$ denote the closure of $\Sigma^I(N, P)$ in $J^r(N, P)$, and ν_I the codimension of $\Sigma^I(N, P)$ in $J^r(N, P)$. Let N be a closed differentiable manifold, $n=\dim N$ and $p=\dim P$. The canonical fiber of $\Sigma^I(N, P)$ over $N\times P$ will be denoted by $\Sigma^I(n, p)$. We will define in § 2 and § 6 the dual class $[\bar{\Sigma}^I(N, P)]$ in $H^{\nu_I}(N\times P; G)$ of the Thom-Boardman singularity $\Sigma^I(N, P)$. The coefficient group G denotes either $\boldsymbol{Z}$ or $\boldsymbol{Z}_2$ depending on whether $\Sigma^I(n, p)$ is orientable or not. When G is $\boldsymbol{Z}$, we assume N and P to be orientable manifolds. For a differentiable map $f: N\to P$ we denote the class $(\mathrm{id}_N\times f)^*([\bar{\Sigma}^I(N, P)])$ in $H^{\nu_I}(N; G)$ by $c^I(TN, f^*(TP))$. We will give in § 5 a formula to calculate the dual class $c^I(TN, f^*(TP))$ in a finite process in terms of the characteristic classes of N and P. The $\boldsymbol{Z}_2$-reduction of these dual classes coincides with those which have been defined in [15] under the sheaf homology (cohomology resp.) groups with closed supports. We will use the singular homology (cohomology resp.) groups in our definition. We will show the following two applications of the dual classes.

Let $\Omega^I(N, P)$ denote the union of all Thom-Boardman singularities with symbol smaller than or equal to I in the lexicographic order. Let $C^\infty_{\Omega^I}(N, P)$ denote the space of all differentiable maps, $f: N\to P$ such that the image of $j^rf: N\to J^r(N, P)$ is contained in $\Omega^I(N, P)$ with C^∞-topology. Let $\Gamma_{\Omega^I}(N)$ denote the space of all continuous sections of the fiber bundle of $J^r(N, P)$ over N such that the image of a section is contained in $\Omega^I(N, P)$ equipped with compact-open topology. Let $\Omega(N, P)$ denote $\Omega^I(N, P)\backslash\Sigma^I(N, P)$ and we consider similarly