J. Math. Soc. Japan
Vol. 34, No. 2, 1982

No explosion criteria for stochastic differential equations

By Kiyomasa NARITA

(Received July 26, 1979)
(Revised July 25, 1980)

§1. Introduction.

In this paper, the existence problem of global solutions of stochastic differential equations will be discussed.

First of all we introduce the notations and definitions. Let I denote the interval $0\leqq t<\infty$ and R^d denote Euclidean d-space. For $x\in R^d$ and $y\in R^d$, let $\langle x, y\rangle$ be the inner product of x and y and let $|x|$ be the Euclidean norm of x. For a $d\times d$-matrix $M=(m_{ij})$, define $|M|=(\sum_{i,j=1}^{d} m_{ij}^2)^{1/2}$. We shall denote by C_2 the family of scalar functions defined on $I\times R^d$ which are twice continuously differentiable with respect to $x\in R^d$ and once with respect to $t\in I$. Let $(\Omega, \boldsymbol{F}, P)$ be a probability space with an increasing family $\{\boldsymbol{F}_t ; t\geqq 0\}$ of sub-σ-algebras of $\boldsymbol{F}$ and let $w(t)=(w_i(t))$, $i=1, \cdots, d$, be a d-dimensional Brownian motion process adapted to $\boldsymbol{F}_t$. Consider the stochastic differential equation

$$(1.1)\qquad dX(t)=b(t, X(t))dt+\sigma(t, X(t))dw(t),$$

where $b(t, x)=(b_i(t, x))$, $i=1, \cdots, d$, is a d-vector function and $\sigma(t, x)=(\sigma_{ij}(t, x))$, $i, j=1, \cdots, d$, is a $d\times d$-matrix function, which are defined on $I\times R^d$ and Borel measurable with respect to the complete set of variables. Equation (1.1) is equivalent to the system of d equations

$$(1.1)'\qquad dX_i(t)=b_i(t, X(t))dt+\sum_{j=1}^{d}\sigma_{ij}(t, X(t))dw_j(t), \qquad i=1, \cdots, d.$$

Throughout this paper, we assume the following:

(1.2) $b(t, x)$ and $\sigma(t, x)$ are continuous in (t, x), and for any $T>0$, $R>0$, there exists a constant $C_{TR}>0$ depending only on T and R such that

$$|b(t, x)-b(t, y)|+|\sigma(t, x)-\sigma(t, y)|\leqq C_{TR}|x-y|$$

if $t\leqq T$, $|x|\leqq R$ and $|y|\leqq R$.

Then, for any natural number n, we can construct functions