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A note on E. Michael's example and rectangular products

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1. Introduction.

Throughout this paper, all spaces considered are to be completely regular.

Pasynkov [6] introduced the notion of rectangular product as follows;

DEFINITION. A product space $X\times Y$ is *rectangular* if every cozero subset of $X\times Y$ is a σ-locally finite union of cozero-set rectangles (i.e. products $U\times V$ of cozero subsets of $X\times Y$).

Rectangularity guarantees the product theorem for covering dimension: i.e. if a product $X\times Y$ is rectangular, then $\dim X\times Y\leqq\dim X+\dim Y$. He has shown in many cases the product is rectangular and asked; Is every product $X\times Y$ rectangular?

This question has been answered negatively by examples of Wage [8] and Przymusiński [7] which do not satisfy the product theorem for covering dimension. As a simpler non-rectangular product, it was announced [6] that V. Zolotarev had proved that (Sorgenfrey line)^2 is not rectangular. As another famous non-normal example, we know the example of E. Michael [3]. In this note we establish the following theorem;

THEOREM. *Let X_A be a Hannerization of a metric space X with respect to a subset A of X. Then $A\times X_A$ is rectangular if and only if $A\times X_A$ is normal if and only if A is F_σ in X.*

As a corollary we obtain that (Michael's line)$\times$(Irrationals) is not rectangular.

It is known [6] normality induces rectangularity in products with a metric factor. At the end of this note we give an example of non-normal rectangular product with a metric factor and we will show that rectangularity cannot be preserved under perfect maps.

2. Rectangularity means normality for product $A\times X_A$.

DEFINITION 1. Let A be a subspace of a space X. The family of all sets of the form $U\cup K$, where U is an open subset of X and $K\subset A$, is a topology on X: The set X with this topology is called a *Hannerization of X with*