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Interpolation, trivial and non-trivial homomorphisms in H^{∞}

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1. Introduction.

Let D be the open unit disc in the complex plane. We assume that the reader is somewhat familiar with the theory of $H^{\infty}=H^{\infty}(D)$ as a function algebra (see [4]) including Hoffman's paper [6] on the parts of H^{∞}.

We recall Carleson's fundamental characterization of interpolating sequences for H^{∞} [1]. In terms of the *pseudo hyperbolic* metric, $\chi(z, w)=|z-w|/|1-\bar{z}w|$, a sequence $\{z_n\}$ in D (possibly finite) is interpolating if it is *uniformly separated*, that is

$$\inf_{n} \prod_{k\neq n} \chi(z_k, z_n)>0 .$$

Garnett [3] found a characterization of interpolating sequences, which is more geometric in nature. It is stated in terms of the concept of a *Carleson measure,* namely, a finite measure μ on D for which there exists a constant K such that $\mu(S_{\theta,h})\leqq Kh$ for every set of the form $S_{\theta,h}=\{re^{i\varphi}: 0<1-r<h,\ |\varphi-\theta| \leqq h/2\}$ (equivalently, for all sufficiently small h).

THEOREM (Garnett). *Let* $\{z_n\}$ *be a separated sequence in D, that is, one for which* $\inf_{n\neq k}\chi(z_n, z_k)>0$. *Let* μ *be the measure on D assigning point mass* $1-|z_n|$ *to* z_n *for each n and* 0 *elsewhere. Then,* $\{z_n\}$ *is interpolating if and only if* μ *is a Carleson measure.*

Note that the condition that $\{z_n\}$ be separated is a natural (and simple) one since every uniformly separated sequence is necessarily separated.

It was Hoffman [5], [6] who characterized the Gleason parts of the maximal ideal space $\mathcal{M}$ of H^{∞} as being either singleton points or analytic discs and who saw the connection with interpolating sequences. A homomorphism in $\mathcal{M}$ is *non-trivial* (its part is an analytic disc) or *trivial* (a one point part) corresponding to whether it lies in the closure in $\mathcal{M}$ of some interpolating sequence or not.

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1) Some of the results of this paper form a portion of this author's doctoral thesis (1978) submitted to the University of California, Santa Barbara.