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Asymptotic wave functions and energy distributions for long-range perturbations of the d'Alambert equation

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Introduction.

In this paper, we study the asymptotic behavior for t (time)$\to\infty$ of acoustic waves which propagate in some inhomogeneous fluids filled in an exterior domain Ω of $\boldsymbol{R}^n$, requiring that the perturbation by inhomogeneous fluids is *long-range*. For each wave, the corresponding asymptotic wave function will be constructed from the initial data (Theorem 3.1), and the asymptotic distribution of the wave energy will be calculated by use of the asymptotic wave function (Theorems 5.1~5.3).

In case of homogeneous fluids, these problems have been studied by Wilcox [8] (cf., also Kitahara [1] and Wilcox [9], where are developed the wave propagation phenomena in anisotropic homogeneous media of strongly propagative class). The principal result there states that each solution $w_0(x, t)$ of the d'Alambert equation

$$\partial_t^2 w_0(x, t)=c^2\Delta w_0(x, t) \quad \text{in } \Omega \tag{0.1}$$

($\partial_t=\partial/\partial t$, Δ is the Laplacian in $\boldsymbol{R}^n$ and $c>0$) is asymptotically equal for $t\to\infty$ to a diverging spherical wave of the form

$$w_0^\infty(x, t)=\frac{1}{\sqrt{2}}\sqrt{c}\,r^{-(n-1)/2}F_0(c^{-1}r-t, \tilde{x}) \quad (r=|x|,\ \tilde{x}=x/|x|), \tag{0.2}$$

where the wave profile $F_0(s, \tilde{x})$, $s\in\boldsymbol{R}$, is calculated from the initial data. Moreover, this result is used to the calculation of the asymptotic energy distributions. In [8] the function (0.2) is called the asymptotic wave function. If $\Omega=\boldsymbol{R}^n$, the profile $F_0(s, \tilde{x})$ is the Radon transform of the initial data, and in general case, it is modified by use of the Møller wave operators in the scattering theory. So, the asymptotic wave function (0.2) is closely related to the translation representation of Lax-Phillips [2].

Now, in inhomogeneous fluids, the propagation of acoustic waves is governed by a perturbed d'Alambert equation of the form