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Closed *-derivations on compact groups

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§1. Introduction and preliminaries.

Unbounded derivations have recently become one of the most important branches of the theory of C^*-algebras. Several authors obtained general results concerning the relation between closed *-derivations and strongly continuous one-parameter groups of *-automorphisms (cf. [**3**], [**4**], [**10**]).

S. Sakai ([**11**, Proposition 1.17]) proved that a non-zero closed derivation δ in $C(T)$ (T: the unit circle) commuting with the rotation group $\{\theta_t\}_{t\in R}$ of T is a scalar multiple of the infinitesimal generator of $\{\theta_t\}_{t\in R}$.

In this paper we present a similar result for closed *-derivations commuting with the left translation group on arbitrary compact groups.

A linear map δ in a C^*-algebra A is said to be a derivation if it satisfies the following condition:

(i) the domain $D(\delta)$ of δ is a dense subalgebra of A and $\delta(fg)=\delta(f)g+f\delta(g)$ $(f, g\in D(\delta))$. A derivation δ is said to be a *-derivation if it satisfies:

(ii) $f\in D(\delta)\Rightarrow f^*\in D(\delta)$ and $\delta(f^*)=\delta(f)^*$.

Throughout this paper, G will denote a compact group. Let $C(G)$ be the C^*-algebra of all complex-valued continuous functions on G. Suppose that $\{g_t\}_{t\in R}$ is a continuous one-parameter subgroup of G. We define $\{\tau_t\}_{t\in R}$ by the equation $\tau_t(f)(x)=f(xg_t)$ $(f\in C(G), x\in G, t\in R)$. Then $\{\tau_t\}_{t\in R}$ is a strongly continuous one-parameter group of *-automorphisms of $C(G)$. Let δ be the infinitesimal generator of $\{\tau_t\}_{t\in R}$. Then it is well-known that δ is a closed *-derivation in $C(G)$ with domain $D(\delta)$ which is a dense *-subalgebra of $C(G)$. Further we define the left translation group $\{L_u\}_{u\in G}$ by the equation $L_u(f)(x)=f(u^{-1}x)$ $(f\in C(G), u, x\in G)$. Then it is clear that $L_u\tau_t=\tau_tL_u$, $L_u(D(\delta))=D(\delta)$ and $L_u\delta=\delta L_u$ $(u\in G, t\in R)$.

Our goal in this paper is to prove that the converse is true, that is, we have the following theorem.

THEOREM. *Let G be a compact group. Suppose that δ is a closed *-derivation in $C(G)$ commuting with the left translation group $\{L_u\}_{u\in G}$, that is, $L_u(D(\delta))=D(\delta)$, $L_u\delta=\delta L_u$ $(u\in G)$. Then there exists a continuous one-parameter subgroup $\{g_t\}_{t\in R}$ of G such that δ is the infinitesimal generator of the strongly continuous*