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Some examples of pseudofree S^1-actions on homotopy $(4m+1)$-spheres

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1. Introduction.

Let S^1 be the circle group. In [**15**] and [**16**], Montgomery and Yang introduced the notion of pseudofree S^1-action (see § 2) and classified all pseudofree S^1-actions on homotopy seven-spheres. Recently, Petrie constructed many pseudofree S^1-actions on homotopy $(4m+3)$-spheres with different isotropy groups and slice representations ([**18**], [**19**]).

In this paper, we construct infinitely many pseudofree S^1-actions on homotopy $(4m+1)$-spheres with the following properties: (i) they are S^1-homotopy equivalent to some fixed linear pseudofree S^1-action φ on S^{4m+1}, (ii) their isotropy groups and slice representations coincide with those of (S^{4m+1}, φ), (iii) their equivariant Pontrjagin classes of the tangent bundles are different from one another.

The method of our construction is due to Petrie [**18**], [**19**] and Hsiang [**7**].

The paper is organized as follows:

In § 2, we state our main theorem precisely. In § 3, we prove a preliminary lemma. In §§ 4 and 5, we consider a quasi-equivalence and S^1-transversality respectively. In § 6, we construct an S^1-normal map. In § 7, we consider a signature of an orbit manifold, which is an obstruction to performing equivariant surgery. In § 8, we prove the main theorem.

2. Notations and the main theorem.

In [**15**] and [**16**], a differentiable action of the circle group S^1 on a compact smooth manifold is said to be *pseudofree* if it is an effective action for which every isotropy group is finite and the set of exceptional orbits is finite but not void. Let M be a compact pseudofree S^1-manifold. Let $S^1/\boldsymbol{Z}_n$ be a singular orbit of M and let V_x be the slice representation space of the isotropy group $\boldsymbol{Z}_n$ at $x \in S^1/\boldsymbol{Z}_n$, where $\boldsymbol{Z}_n$ denotes the cyclic group of order n. We remark that the equivalent class $\{V_x\}$ is independent of the choice of $x \in S^1/\boldsymbol{Z}_n$. So we can define an invariant $I(M)$ of M by