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A generalization of Roberts-Tannaka duality theorem

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1. Introduction.

Let $\{\mathfrak{M}, G, \gamma\}$ be a covariant system, that is, G is a locally compact group and $\gamma: G \rightarrow \mathrm{Aut}(\mathfrak{M})$ is a homomorphism of G into the group of $*$-automorphisms of a von Neumann algebra $\mathfrak{M}$ with the following continuity: $G \ni t \rightarrow \gamma_t x \in \mathfrak{M}$ is continuous for each $x \in \mathfrak{M}$ with respect to the σ-weak topology on $\mathfrak{M}$. By definition in [**4**], a *Hilbert space in* $\mathfrak{M}$ is a closed subspace $\mathfrak{K}$ of $\mathfrak{M}$ such that

(i) y^*x is a scalar multiple of the identity for every $x, y \in \mathfrak{K}$ and

(ii) for every non-zero $a \in \mathfrak{M}$, there exists an $x \in \mathfrak{K}$ with $ax \neq 0$.

The inner product $(x|y)$ in $\mathfrak{K}$ is given by y^*x. If a *Hilbert space* $\mathfrak{K}$ *in* $\mathfrak{M}$ is globally invariant under γ, $\gamma_t(\mathfrak{K}) \subseteqq \mathfrak{K}$ for all $t \in G$, we have

$$(\gamma_t x | \gamma_t y) = \gamma_t(y^*x) = y^*x = (x|y) \qquad \text{for every} \quad x, y \in \mathfrak{K}, t \in G.$$

Hence the restriction of γ to $\mathfrak{K}$ is a unitary representation of G. We denote it by $\pi_{\mathfrak{K}}$. Let $\mathcal{H}_\gamma(\mathfrak{M})$ be the collection of all *Hilbert spaces in* $\mathfrak{M}$ globally invariant under γ. Let $\mathfrak{M}^\gamma$ denote the fixed point algebra $\{x \in \mathfrak{M}; \gamma_t(x) = x$ for all $t \in G\}$ of $\mathfrak{M}$ under γ and $\mathrm{Aut}(\mathfrak{M}|\mathfrak{M}^\gamma) = \{\rho \in \mathrm{Aut}(\mathfrak{M}); \rho(x) = x$ for all $x \in \mathfrak{M}^\gamma\}$.

Under the above situation the following Roberts-Tannaka duality theorem was obtained and was used as a basic tool in [**1**].

THEOREM 1. *Assume that* $\mathfrak{M}^\gamma$ *is properly infinite and* G *is compact. If each irreducible subrepresentation of* $\{\gamma, \mathfrak{M}\}$ *is unitarily equivalent to some* $\pi_{\mathfrak{K}}$, $\mathfrak{K} \in \mathcal{H}_\gamma(\mathfrak{M})$, *then every* $\sigma \in \mathrm{Aut}(\mathfrak{M}|\mathfrak{M}^\gamma)$ *leaving every member* $\mathfrak{K} \in \mathcal{H}_\gamma(\mathfrak{M})$ *globally invariant must be of the form* γ_s *for some* $s \in G$.

In this short note we generalize the above theorem to the case of arbitrary locally compact groups. This problem is suggested in [**3**].

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2. A duality theorem.

Before stating the theorem, we show the following lemma.

LEMMA. *If* $\sigma \in \mathrm{Aut}(\mathfrak{M}|\mathfrak{M}^\gamma)$ *and* $\mathfrak{K} \in \mathcal{H}_\gamma(\mathfrak{M})$ *which is globally invariant under* σ, *each globally* γ*-invariant closed subspace* $\mathfrak{K}'$ *of* $\mathfrak{K}$ *is also globally invariant*