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Nonlinear ergodic theorems and weak convergence theorems

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Introduction.

In this paper we study the asymptotic behavior of nonexpansive mappings and of one parameter semigroups of nonexpansive mappings in Banach spaces. In [1], Baillon proved the first nonlinear ergodic theorem for nonexpansive mappings in Hilbert spaces. Reich [16] extended Baillon's result to uniformly convex Banach spaces which have Fréchet differentiable norms and Bruck [8] simplified the original argument of Reich. The weak convergence of trajectories of one parameter semigroups of nonexpansive mappings was studied by Baillon [2], Bruck [7], Pasy [16], Miyadera [12] and Reich [17]. In section 2, we give ergodic theorems for nonexpansive mappings in uniformly convex Banach spaces which satisfy Opial's condition. In section 3, we consider a necessary and sufficient condition for the weak convergence of trajectories of nonexpansive mappings and one parameter semigroups of nonexpansive mappings in Banach spaces.

1. Preliminaries and notations.

Let C be a closed convex subset of a Banach space E. A mapping $T: C\rightarrow E$ is said to be nonexpansive if

$$\|Tx-Ty\|\leqq\|x-y\| \qquad \text{for all} \quad x, y\in C.$$

A one parameter semigroup $S=\{S(t): t\geqq 0\}$ of nonexpansive mappings on C is a family of nonexpansive mappings of C into itself satisfying the following conditions

(1.1) $\quad S(s+t)x=S(s)S(t)x \qquad$ for $s, t\geqq 0$ and $x\in C$;

(1.2) $\quad \|S(t)x-S(t)y\|\leqq\|x-y\| \qquad$ for $t\geqq 0$ and $x, y\in C$;

(1.3) $\quad S(0)x=x \qquad$ for $x\in C$;

(1.4) $\quad \lim_{t\rightarrow t_0} S(t)x=S(t_0)x \qquad$ for $t, t_0\geqq 0$ and $x\in C$.