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Segal-Becker theorem for KR-theory

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§1. Introduction.

The purpose of this paper is to prove the following result.

THEOREM 1.1. *Let X be a based finite $\boldsymbol{Z}_2$-complex in the sense of* [5]. *Then there exists a natural split epimorphism*

$$\lambda_*: \{X, \boldsymbol{C}P^\infty\}_{Z_2} \longrightarrow \tilde{K}_R(X).$$

As corollaries of this theorem we deduce the results of G. Segal [12] and J. C. Becker [4].

First we fix our notation.

Let X be a compact based $\boldsymbol{Z}_2$-space and let

$$\tilde{K}_R(X)=K_R(X, *)$$

be the reduced KR-group of Atiyah [2]. If X and Y are based $\boldsymbol{Z}_2$-spaces, then $[X, Y]_{Z_2}$ denotes the set of $\boldsymbol{Z}_2$-homotopy classes of based $\boldsymbol{Z}_2$-maps from X to Y. Let $\boldsymbol{R}^{p,q}$ be the representation of $\boldsymbol{Z}_2$ on $\boldsymbol{R}^{p+q}$ given by

$$g(x_1, \cdots, x_{p+q})=(-x_1, \cdots, -x_p, x_{p+1}, \cdots, x_{p+q}), \qquad g\in \boldsymbol{Z}_2,$$

and let $\Sigma^{p,q}=(\boldsymbol{R}^{p,q})^c$ be the one-point compactification of $\boldsymbol{R}^{p,q}$. Then we define the stable $\boldsymbol{Z}_2$-homotopy group $\{X, Y\}_{Z_2}$ to be $\varinjlim_n [\Sigma^{n,n}\wedge X, \Sigma^{n,n}\wedge Y]_{Z_2}$. Let $\boldsymbol{C}P^n$ be the complex projective space with the involution σ given by

$$\sigma[z_0, \cdots, z_n]=[\bar{z}_0, \cdots, \bar{z}_n].$$

The construction of λ_* is given as follows.

Let BR denote a classifying space for stable Real vector bundles so that $\tilde{K}_R(X)=[X, BR]_{Z_2}$. (We shall give a specific model for BR in §3.) For a $\boldsymbol{Z}_2$-space X,

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