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Free boundary problems for a class of nonlinear parabolic equations: An approach by the theory of subdifferential operators

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Introduction.

The present paper is devoted to the study of a free boundary problem for a nonlinear parabolic equation in one-space dimension. Free boundary problems arise naturally in a number of physical phenomena with change of state (such as melting of ice and recrystallization of metals) and have been studied by many authors (e.g., [**1**, **2**, **4-10**, **14-19**] and their references).

In this paper we are concerned with the following one phase Stefan problem: For a number $l_0 \geqq 0$, functions u_0 on $[0, l_0]$, f on $[0, T]\times[0, \infty)$ and g on $[0, T]$ we find a boundary curve $x=l(t)$ ($\geqq 0$ on $[0, T]$) and a function $u=u(t, x)$ on $[0, T]\times[0, \infty)$ satisfying

$$\text{(E)} \qquad u_t-(|\beta(u)_x|^{p-2}\beta(u)_x)_x=f \qquad \text{for } l(t)>0,\ 0<x<l(t)$$

subject to

$$\text{(C1)} \qquad l(0)=l_0 \text{ and if } l_0>0, \quad \text{then } u(0, x)=u_0(x) \quad \text{for } 0<x<l_0,$$

$$\text{(C2)} \qquad \begin{cases} |\beta(u)_x(t, 0+)|^{p-2}\beta(u)_x(t, 0+)=g(t) & \text{for } 0<t<T, \\ \beta(u)(t, l(t))=0 & \text{for } 0<t<T \end{cases}$$

and

$$\text{(C3)} \qquad \frac{dl(t)}{dt}=-|\beta(u)_x(t, l(t)-)|^{p-2}\beta(u)_x(t, l(t)-) \qquad \text{for } 0<t<T,$$

where $2\leqq p<\infty$, $\beta: \boldsymbol{R}\to\boldsymbol{R}$ is a given function and $\beta(u)_x(t, x+)$ (resp. $\beta(u)_x(t, x-)$) stands for the right (resp. left) hand partial derivative of $\beta(u)(t, x)$ at x with respect to x.

This kind of problems for a certain class of nonlinear parabolic equations was treated earlier by Douglas [**6**] and Kyner [**16**] in which they showed the existence and uniqueness of solution by using a strong maximum principle for parabolic equations with variable coefficients, but their method is not applicable to our case. Our approach to problem {(E), (C1)-(C3)}, which is different from that of Douglas and Kyner in some points of view, is based upon recent results