J. Math. Soc. Japan
Vol. 33, No. 4, 1981

Eigenvalues of the Laplacian on Calabi-Eckmann manifolds

By Kazumi TSUKADA

(Received March 24, 1980)

1. Introduction.

On a compact Hermitian manifold M, we define two differential operators, i.e., the real Laplacian $\triangle = d\delta + \delta d$, and the complex Laplacian $\square = \bar{\partial}\theta + \theta\bar{\partial}$. In this note we deal with these operators acting on differentiable functions. We denote by $\mathrm{Spec}(M, \triangle)$ (resp. $\mathrm{Spec}(M, \square)$) the set of eigenvalues with multiplicity of $\triangle$ (resp. $\square$). It is an interesting problem to investigate the relationship between the geometry of a smooth manifold and the spectrum of its Laplacian.

A Hopf manifold is well-known as the first example of a compact complex manifold which does not admit Kaehler metrics. E. Bedford and T. Suwa ([**1**]) described explicitly the eigenvalues of $\triangle$ and $\square$ on Hopf manifolds. Some isospectral results were also given by them. In this note we will describe (in Theorem 5.7) the eigenvalues of $\triangle$ and $\square$ on Calabi-Eckmann manifolds which were discovered as the second example of non-Kaehler complex manifolds ([**4**]). Some isospectral results will be given in Theorem 6.4 and Theorem 6.5.

The author wishes to thank Professor K. Ogiue for helpful suggestions.

2. The complex Laplacian on Hermitian manifolds.

In this section we will find the relation between the complex and the real Laplacians, and give formulas for the asymptotic expansion of the eigenvalues of the complex Laplacian, making use of Gilkey's Theorem.

Let (M, g, J) be a compact connected Hermitian manifold with Hermitian metric g and complex structure J. By $C^\infty(M)$ we denote the space of complex-valued differentiable functions on M with a scalar product $\langle\varphi, \phi\rangle = \int_M \varphi\bar{\phi}\, dV_g$, where dV_g is the Riemannian volume form on M. For the definition and the fundamental properties of the complex Laplacian on Hermitian manifolds we refer to Morrow and Kodaira [**9**]. Let Ω be the 2-form defined by $\Omega(X, Y) = g(JX, Y)$, which is called the *Kaehler form* of (M, g, J). Let ζ be the vector field dual to $-\delta\Omega$, where δ denotes the codifferential operator.