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A note on Yoneda product

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1. Introduction.

Let G be a group, Z the ring of integers, m a positive integer and Z_m the ring of integers modulo m. It is well known ([5], Proposition 5) that Yoneda product in the cohomology ring $\mathrm{Ext}^*_{Z_mG}(Z_m, Z_m)$ is anti-commutative. The aim of the present note is to prove that this anti-commutative property does not hold in the cohomology ring $\mathrm{Ext}^*_{ZG}(Z_m, Z_m)$. Recall that $a, b \in \mathrm{Ext}^*(A, A)$ of degree r, s respectively are said to anti-commute if $ab=(-1)^{r+s}ba$.

2. Preliminaries.

Let G, Z, m and Z_m be as in the introduction. The exact sequence

$$0 \longrightarrow Z \xrightarrow{m} Z \xrightarrow{\alpha} Z_m \longrightarrow 0 \tag{2.1}$$

of trivial G-modules where α is the natural projection determines an element e of $\mathrm{Ext}^1_{ZG}(Z_m, Z)$ ([4], pp. 84-85; [3], p. 494). For ZG-modules A, B the connecting homomorphisms

$$\delta^r : \mathrm{Ext}^r_{ZG}(A, Z_m) \longrightarrow \mathrm{Ext}^{r+1}_{ZG}(A, Z) \quad \text{and}$$

$$\partial^s : H^s(G, B) \longrightarrow \mathrm{Ext}^{s+1}_{ZG}(Z_m, B)$$

are then given by ([3], p. 493)

$$\delta^r(a) = -ea\,, \quad a \in \mathrm{Ext}^r_{ZG}(A, Z_m) \quad \text{and}$$

$$\partial^s(b) = be\,, \quad b \in H^s(G, B)\,.$$

Here the product involved is the Yoneda product and observe that if $x \in \mathrm{Ext}^r_{ZG}(A, B)$, $y \in \mathrm{Ext}^s_{ZG}(B, C)$, then $yx \in \mathrm{Ext}^{r+s}_{ZG}(A, C)$.

For a ZG-module B, let

$$R(B): 0 \longrightarrow B \xrightarrow{\varepsilon_B} R^0(B) \xrightarrow{d^0_B} R^1(B) \xrightarrow{d^1_B} \cdots \longrightarrow R^n(B) \xrightarrow{d^n_B} R^{n+1}(B) \longrightarrow \cdots$$