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On expansive homeomorphisms on manifolds

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1. Introduction.

X will be a metric space with a metric d. A homeomorphism f of X onto itself is expansive if there exists a positive number C (called expansive constant) such that for each pair (x, y) of distinct points of X, there is an integer n for which $d(f^{n}(x), f^{n}(y))>C$.

There is a question what manifolds admit such homeomorphisms. Several examples of existence and non-existence of expansive homeomorphisms are known. An open interval, a 1-sphere and a closed 2-disk do not admit expansive homeomorphisms (Bryant [**1**], Jakobsen and Utz [**2**]). An open $2n$-ball ($n\geqq 1$) and an r-dimensional torus ($r\geqq 2$) admit expansive homeomorphisms (Reddy [**3**]). In this paper, we prove the followings.

THEOREM 1. *Let M be a closed n-manifold* ($n\geqq 1$), *and J be an open interval. Then there exists an expansive homeomorphism of $M\times J$.*

THEOREM 2. *If M is a closed n-manifold* ($n\geqq 1$), *there exist an expansive homeomorphism of* $\mathrm{Int}(M^{*}\{point\})$. *Where $P^{*}Q$ is the join of P and Q, and* $\mathrm{Int}\, M$ *is the interior of M.*

COROLLARY. *There exists an expansive homeomorphism of an open n-ball* ($n\geqq 2$).

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2. Proof of Theorem 1.

Let M be a closed n-manifold with a metric d. $J=(0, 2)$ and R^{n} be an open interval with a standard metric d_{1} and an n-dimensional Euclidean space with a standard metric d_{n}, respectively. And put $U(x, \varepsilon)=\{y\in M \mid d(y, x)<\varepsilon\}$, $U_{n}(z, \delta)=\{y\in R^{n} \mid d_{n}(y, z)<\delta\}$. We define the metric ρ of $M\times J$ to be $d\times d_{1}$ (where $d\times d_{1}((x, t), (y, s))=d(x, y)+d_{1}(t, s)$ and $x, y\in M$ and $t, s\in J$), and I_{k} ($k\geqq 0$) to be $I_{k}=\left[\frac{1}{k+1}, \frac{1}{k}\right]$ ($k\in N$) and $I_{0}=[1, 2)$. Put $A_{k}=M\times I_{k}$.

First, we define several homeomorphisms of A_{1}. We will use these homeomorphisms for constructing an expansive homeomorphism of $M\times J$. For any