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# Comparison theorems for functional differential equations with deviating arguments

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## Introduction.

We consider the functional differential equations with deviating arguments

$(L_n^+, F, g)$ $$L_n x(t)+F(t, x(g(t)))=0,$$

$(L_n^-, F, g)$ $$L_n x(t)-F(t, x(g(t)))=0,$$

where $n \geqq 2$ and $L_n$ denotes the disconjugate differential operator

(1) $$L_n=\frac{1}{p_n(t)}\frac{d}{dt}\frac{1}{p_{n-1}(t)}\frac{d}{dt}\cdots\frac{d}{dt}\frac{1}{p_1(t)}\frac{d}{dt}\frac{\cdot}{p_0(t)}.$$

We always assume that:

(L-1) $p_i, g:[a, \infty)\to R$ are continuous, $p_i(t)>0$, $0\leqq i\leqq n$, and $g(t)\to\infty$ as $t\to\infty$;

(L-2) $F:[a, \infty)\times R\to R$ is continuous, and $\operatorname{sgn} F(t, x)=\operatorname{sgn} x$ for each $t\in[a, \infty)$.

We introduce the notation:

(2) $$D^0(x; p_0)(t)=\frac{x(t)}{p_0(t)},$$
$$D^i(x; p_0, \cdots, p_i)(t)=\frac{1}{p_i(t)}\frac{d}{dt}D^{i-1}(x; p_0, \cdots, p_{i-1})(t), \quad 1\leqq i\leqq n.$$

The operator $L_n$ can then be rewritten as

$$L_n=D^n(\cdot; p_0, \cdots, p_n).$$

The domain $\mathcal{D}(L_n)$ of $L_n$ is defined to be the set of all functions $x:[T_x, \infty)\to R$ such that $D^i(x; p_0, \cdots, p_i)$, $0\leqq i\leqq n$, exist and are continuous on $[T_x, \infty)$. By a proper solution of equation $(L_n^+, F, g)[(L_n^-, F, g)]$ is meant a function $x\in\mathcal{D}(L_n)$ which satisfies $(L_n^+, F, g)[(L_n^-, F, g)]$ for all sufficiently large $t$ and $\sup\{|x(t)|: t\geqq T\}>0$ for every $T\geqq T_x$. We make the standing hypothesis that equations