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On G-extensible regularity condition and Thom-Boardman singularities

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0. Introduction.

In [2], we have defined a G-extensible regularity condition on equivariant sections of differentiable G-fibre bundle P. In this paper, we only consider the case where P is a trivial G-fibre bundle as an application of Theorem 1.3 in [2].

We now formulate as follows: Let G be a compact Lie group. Let X, Y be smooth G-manifolds. Then the r-jet bundle $J^r(X, Y)$ is naturally a differentiable G-fibre bundle such that the action of G on $J^r(X, Y)$ is defined by $g(j_x^r f)=j_{gx}^r(gfg^{-1})$ where $g\in G$ and f is a germ of a map $X\rightarrow Y$ at $x\in X$. Let $J_G^r(X, Y)$ be the subspace of $J^r(X, Y)$ consisting of r-jets of "equivariant local maps" $X\rightarrow Y$. Then $J_G^r(X, Y)$ is a G-invariant subspace of $J^r(X, Y)$.

Now let $\Omega(X, Y)$ be an open G-subbundle of $J^r(X, Y)\rightarrow X$ invariant under the natural action by local equivariant diffeomorphism of X on $J^r(X, Y)$. Then $\Omega(X, Y)$ is called a *natural stable regularity condition.*

We shall say that a map $f: X\rightarrow Y$ is *Ω-regular* if $j^rf(X)\subset\Omega(X, Y)$.

DEFINITION 0.1. Let $\Omega(X, Y)$ be a natural stable regularity condition. We say that $\Omega(X, Y)$ is *G-extensible* if the following conditions hold:

There exists a natural stable regularity condition $\Omega'(X\times \boldsymbol{R}, Y)\subset J^r(X\times \boldsymbol{R}, Y)$ (where G acts on $\boldsymbol{R}$ trivially) such that

$$\begin{cases} \pi(i^*(\Omega'(X\times \boldsymbol{R}, Y)))=\Omega(X, Y) \\ \pi(i^*(\Omega'(X\times \boldsymbol{R}, Y)\cap J_G^r(X\times \boldsymbol{R}, Y)))=\Omega(X, Y)\cap J_G^r(X, Y), \end{cases}$$

where $\pi: i^*(J^r(X\times \boldsymbol{R}, Y))\rightarrow J^r(X, Y)$ is defined by $\pi(j_{(x,0)}^r f)=j_x^r fi$ for the canonical inclusion $i: X\hookrightarrow X\times \boldsymbol{R}$. (We call that $\Omega'(X\times \boldsymbol{R}, Y)$ is the *extension* of $\Omega(X, Y)$).

From [2], we have the following theorem.

THEOREM 0.2. *Let $C_{G\Omega}^\infty(X, Y)$ be the space of the Ω-regular equivariant maps $X\rightarrow Y$, with the C^∞-topology, and let $\Gamma_G^0(\Omega_G(X, Y))$ be the space of continuous equivariant sections of the map $\Omega(X, Y)\cap J_G^r(X, Y)\rightarrow X$ (with the compact-open topology). Then, if $\Omega(X, Y)$ is G-extensible,*

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