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Nonselfadjoint crossed products II

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1. Introduction.

This paper is a continuation of [7]. We are interested in the invariant subspace structure and ideal structure of certain subalgebras of von Neumann algebras constructed as crossed products of finite von Neumann algebras by trace preserving automorphisms. These subalgebras are called nonselfadjoint crossed products and most properly should be regarded as operator theoretic versions of twisted polynomial rings. We seek conditions under which an analogue of the theorem of Beurling (as generalized by Lax and Halmos) is valid. The theorem of Beurling, Lax and Halmos (hereafter abbreviated the BLH theorem) is usually regarded as describing the invariant subspaces of a unilateral shift (of arbitrary multiplicity). However, from a ring theoretic point of view it may be thought of as describing certain modules over the algebra $H^{\infty}(\varDelta)$ of bounded analytic functions on the unit disc (regarded as a subalgebra of L^{∞} of the circle), and in particular the BLH theorem implies that every weak*-closed ideal in $H^{\infty}(\varDelta)$ is principal. Thus, from an operator theoretic point of view $H^{\infty}(\varDelta)$ is a principal ideal domain, a generalization of the polynomial algebra in one variable. Since von Neumann algebra crossed products may be viewed as noncommutative generalizations of L^{∞} of the circle and since our nonselfadjoint crossed products are generalizations of $H^{\infty}(\varDelta)$, our search for analogues of the BLH theorem is tantamount to looking for conditions under which our algebras are noncommutative principal ideal rings. We shall find necessary and sufficient conditions for the validity of the BLH theorem for a nonselfadjoint crossed product and we shall prove that within the context of subdiagonal algebras defined and first studied by Arveson in [1], the validity of the BLH theorem essentially characterizes nonselfadjoint crossed products. More precisely, we shall show that with a minor qualification if a subdiagonal algebra has the property that every ultraweakly closed two-sided ideal is principal, then the algebra is a nonselfadjoint crossed product and every ultraweakly closed left ideal is principal.

The setting here is the following. Let M be a von Neumann algebra with

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