J. Math. Soc. Japan
Vol. 32, No. 2, 1981

Recurrence properties of Lotka-Volterra models with random fluctuations

By H. KESTEN[1) and Y. OGURA

(Received July 16, 1979)

1. Introduction and statement of results.

A number of authors ([6], [14], [15]) have recently considered the (Ito) stochastic equation

$$
\begin{aligned}
(1.1)\qquad dX_i(t)=&X_i(t)\{a_{i1}dW_1+a_{i2}dW_2\}\\
&+X_i(t)\{k_i-b_{i1}X_1^{\theta_1}(t)-b_{i2}X_2^{\theta_2}(t)\}\,dt\,, \qquad i=1,2
\end{aligned}
$$

on the first quadrant $Q\equiv\{X_1>0,\ X_2>0\}$. Here a_{ij}, b_{ij}, k_i and θ_i are constants which satisfy

$$
(1.2)\qquad a_{11}a_{22}-a_{12}a_{21}\neq 0\,,\qquad \theta_i>0\,.
$$

The interest in these equations arises from their interpretation as a description of a system of two competing species or a predator-prey model in a randomly varying environment. In this interpretation $X_i(t)$ represents the amount of species i present at time t. Turelli [15] gives a thorough discussion of the validity of this interpretation. Even though there are difficulties in justifying (1.1) as the correct model for competing species in a randomly varying environment, it is believed that its solution behaves similar to real systems as far as absorption and explosion is concerned. In this paper we take (1.1) for granted and discuss the question of recurrence or transience of the system.

Specifically, we introduce

$$
\begin{aligned}
&\xi'_M=\inf\{t\geqq 0:\ |X(t)|\geqq M\}\,,\\
&\xi''_M=\inf\{t\geqq 0:\ X_1(t)\leqq M^{-1}\ \text{or}\ X_2(t)\leqq M^{-1}\}\,,\\
&\xi'=\lim_{M\to\infty}\xi'_M\,,\qquad \xi''=\lim_{M\to\infty}\xi''_M\,.
\end{aligned}
$$

ξ' and ξ'' are called the explosion time, respectively absorption time. For $X(0)\in Q$ fixed the solution of (1.1) is unique up till time $\xi'\wedge\xi''$ ([5] Theorem

1) The first author was supported by the NSF through a grant to Cornell University.