J. Math. Soc. Japan
Vol. 33, No. 2, 1981

Bounds on the degree of the equations defining Kummer varieties

By Ryuji SASAKI

(Received July 16, 1979)

Let k be an algebraically closed field whose characteristic is not equal to two. Let K be the Kummer variety of an abelian variety X over k, i.e., the quotient of X by the inverse morphism $\iota: X\rightarrow X$, and let M be an ample invertible sheaf on K. For any positive integer a, we denote by $\Phi_{M^a}: K\rightarrow \boldsymbol{P}(\Gamma(K, M^a))$ the mapping defined by the linear system $\Gamma(K, M^a)$. In Section 1, we shall prove the projective normality of Kummer varieties (Corollary 1.5):

The image $\Phi_{M^a}(K)$ is projectively normal for any $a\geqq 2$; moreover if the canonical mapping

$$\Gamma(K, M)\otimes\Gamma(K, M)\rightarrow\Gamma(K, M^2)$$

is surjective, then the image $\Phi_M(K)$ is also projectively normal.

In the last section 2, we shall prove the main result (Theorem 2.1) in the present paper, which asserts:

The image $\Phi_{M^a}(K)$ is (set-theoretically) an intersection of cubics when $a=2$ and quadrics when $a\geqq 3$.

This gives a partial answer to a problem proposed by D. Mumford.

I want to thank Professor S. Koizumi and Dr. T. Sekiguchi for their encouragement and suggestion during the preparation of this paper, especially the latter's suggestion was very useful for proving the main result.

NOTATION AND TERMINOLOGY

Throughout this paper k is an algebraically closed field of characteristic $p\neq 2$. X will denote an abelian variety over k of dimension g. For an integer n, X_n is the kernel of the homomorphism $n_X: X\rightarrow X$ defined by $x\mapsto nx$. Let L be an invertible sheaf on X. Then we denote by $K(L)$ the kernel of the homomorphism $\phi_L: X\rightarrow\hat{X}$ of X to the dual of X defined by $x\mapsto T_x^*L\otimes L^{-1}$, and denote by $\mathcal{G}(L)$ the theta group of L. As usual the action of $\mathcal{G}(L)$ is denoted by U. $P=P_X$ will denote the Poincaré invertible sheaf on $X\times\hat{X}$, and P_α is the restriction of P to $X\times\{\alpha\}$ for any point α of $\hat{X}$. For a linear form f on a finite dimensional vector space V over k, we denote by $[f]$ the point in the projective space $\boldsymbol{P}(V)$ determined by f.