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A negative answer to a conjecture of conformal transformations of Riemannian manifolds

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1. Introduction.

Let (M, g), or simply M, be an n-dimensional differentiable manifold with Riemannian metric g. We denote by $C_0(M, g)$ the largest connected group of conformal transformations of (M, g), and by $I_0(M, g)$ the largest connected group of isometries of (M, g).

Riemannian manifolds with constant scalar curvature admitting an infinitesimal non-isometric conformal transformation have been extensively studied by various authors, and the following conjecture has been well-known.

CONJECTURE. *Let (M, g) be an n-dimensional compact Riemannian manifold. If*

(i) $n>2$

(ii) *the scalar curvature of (M, g) is constant*

(iii) $C_0(M, g)\neq I_0(M, g)$,

then (M, g) is isometric to a Euclidean n-sphere S^n.

This conjecture has been proved in various forms under some stronger assumptions. Typical results may be quoted as follows.

THEOREM A (Yano and Nagano [8]). *The conjecture is true if, instead of* (ii),

$(ii)_A$ *(M, g) is Einstein.*

THEOREM B (Nagano [6]). *The conjecture is true if, instead of* (ii),

$(ii)_B$ *the Ricci tensor of (M, g) is parallel.*

THEOREM C (Goldberg and Kobayashi [2], [3]). *The conjecture is true if, instead of* (i) *and* (ii),

$(i)_C$ $n>3$

$(ii)_C$ *$I_0(M, g)$ is transitive on M.*

THEOREM D (Lichnerowicz [5]). *The conjecture is true if instead of* (ii),

$(ii)_D$ *the scalar curvature and the length of the Ricci tensor of (M, g) are constant.*

THEOREM E (Hsiung [4]). *The conjecture is true if, instead of* (ii),

$(ii)_E$ *the scalar curvature and the length of curvature tensor of (M, g) are con-*