J. Math. Soc. Japan
Vol. 33, No. 2, 1981

Weierstrass points on compact Riemann surfaces with nontrivial automorphisms

By Nobumasa TAKIGAWA

(Received June 13, 1979)

1. Introduction.

Let S be a compact Riemann surface of genus g ($\geqq 3$), h be an automorphism of S with fixed points, and T denotes the number of these fixed points. Let $\langle h\rangle$ denote the cyclic group generated by h, whose order is an odd prime number p. Let $S/\langle h\rangle$ be the surface obtained by identifying the equivalent points on S under the elements of $\langle h\rangle$. If $S/\langle h\rangle$ has genus zero, then S can be defined by an equation of the form

$$y^{p}=\prod_{j=1}^{T}(x-c_{j})^{\delta_{j}}, \tag{1}$$

where c_j $(1\leqq j\leqq T)$ are complex numbers which are different from each other, $1\leqq\delta_j\leqq p-1$, and $\sum_{j=1}^{T}\delta_j\equiv 0 \pmod p$. Throughout the present paper we consider only these surfaces. We show that they are characterized by non-negative integral solution (under suitable conditions) of the system of linear equations, which are derived from J. Lewittes' method [**4**]. We investigate also the Weierstrass gap sequence at the point Q_j on S corresponding to $(c_j, 0)$.

A matrix representation $R_1(h)$ of $\langle h\rangle$ is obtained by letting it act on the complex g-dimensional space $A_1(S)$ of Abelian differentials of the first kind. Let n_k with $0\leqq k\leqq p-1$ denote the multiplicity of ε^k ($\varepsilon=\exp\{(2\pi i)/p\}$) in the diagonal form of $R_1(h)$. The upper (resp. lower) bound of $\{n_k\}$ is taken over all compact Riemann surfaces of fixed genus g with an automorphism h which satisfy properties mentioned above. This upper (resp. lower) bound we call simply the upper (resp. lower) bound of $\{n_k\}$, and it is denoted by n^* (resp. n_*). Lewittes has given upper and lower bound of $\{n_k\}$ if $T>0$, [**4**, Theorem 4 (c)]. Our bounds given in this paper are ones improved on Lewittes' results except for $T\equiv 0 \pmod p$. In section 4, we consider the condition (A_0), and show that an automorphism h satisfies the condition (A_0) with respect to λ $(1\leqq\lambda\leqq p-1)$ if and only if $n_\lambda=n^*$ (resp. $n_{p-\lambda}=n_*$). This condition (A_0) contains the Kato's condition (A), [**3**, p. 398]. Kato has shown that if an automorphism