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On infinite dimensional unitary representations of certain discrete groups

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§ 0. Introduction.

0.0. For the modular group $SL_2(\boldsymbol{Z})$, M. Saito [7] has constructed certain series of infinite dimensional unitary representations by classifying and decomposing the representations induced from unitary characters of Cartan subgroups of $SL_2(\boldsymbol{Z})$. The purpose of this note is to make a few remarks which either clarify the interconnection or generalize the results of Saito's construction.

0.1. Let G be a group, and $\mathcal{A}$ a family of subgroups of G. The pair $(G, \mathcal{A})$ is said to have Property $(\mathcal{F})$, if the following two requirements are fulfilled.

($\mathcal{F}$1) For $H_1, H_2 \in \mathcal{A}$, and $g \in G$,
$[H_1 : H_1 \cap g^{-1}H_2 g] < \infty \Rightarrow H_1 \subset g^{-1}H_2 g$.

($\mathcal{F}$2) For $H \in \mathcal{A}$, and $g \in G$,
$g^{-1}Hg \subset H \Rightarrow g^{-1}Hg = H$.

Now, suppose moreover that G is a locally compact topological group and any member H_i of $\mathcal{A}$ is an open subgroup of G. Let χ_i be an irreducible unitary representation of H_i and let $U_i = \mathrm{Ind}(\chi_i : H_i \uparrow G)$ denote the representation of G induced by χ_i. The points of [7] can be summarized in the following (I)~(IV).

(I) Assume that χ_i is one dimensional, then the following three conditions are mutually equivalent (Théorème 2 [7]).

(i) U_1 is equivalent to U_2.

(ii) U_1 is not disjoint from U_2.

(iii) There exists $g \in G$ such that $H_2 = g^{-1}H_1 g$ and $\chi_2 = {}^g\chi_1$, where ${}^g\chi_1(x) = \chi_1(gxg^{-1})$ for $x \in H_2$.

(II) If U_1 is not disjoint from U_2 (hence we may assume $H_1 = H_2 = H$ and $\chi_1 = \chi_2 = \chi$, and put $N_\chi = \{g \in N_G(H) \mid {}^g\chi = \chi\}$), then the dimension of the space of all intertwining operators of $U(\chi) = \mathrm{Ind}(\chi : H \uparrow G)$ is given by the group index $[N_\chi : H]$ (Théorème 1 [7]).

(III) If $G = SL_2(\boldsymbol{Z})$ and $\mathcal{A}$ is the set of all Cartan subgroups of G, then the pair $(G, \mathcal{A})$ has Property $(\mathcal{F})$.