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Regular embeddings of C^*-algebras in monotone complete C^*-algebras

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Introduction.

Let A be a unital C^*-algebra and $A_{s.a.}$ the self-adjoint part of A. If each bounded increasing net (resp. sequence) in $A_{s.a.}$ has a supremum then A is said to be *monotone* (resp. *monotone* σ-) *complete*. [In the literature, e. g., [**10**, **16**, **20**], the adjective "monotone (resp. monotone σ-) closed" is employed as a synonym for "monotone (resp. monotone σ-) complete", but in this paper we will use it in a different sense (cf. Definition 1.2).] As was shown by J. D. M. Wright [**22**], each unital C^*-algebra A possesses a unique *regular σ-completion*, i. e., a monotone σ-complete C^*-algebra $\hat{A}$ which contains A as a C^*-subalgebra and satisfies the following properties:

i) $\hat{A}_{s.a.}$ itself is a unique monotone σ-closed subspace of $\hat{A}_{s.a.}$ which contains $A_{s.a.}$;

ii) each x in $\hat{A}_{s.a.}$ is the supremum in $\hat{A}_{s.a.}$ of $\{a\in A_{s.a.}: a\leqq x\}$; and

iii) whenever a subset $\mathscr{F}$ of $A_{s.a.}$ has a supremum x in $A_{s.a.}$ then x remains the supremum of $\mathscr{F}$ in $\hat{A}_{s.a.}$.

On the other hand the present author proved in [**6**] that each unital C^*-algebra A has a unique *injective envelope*, which will be written as $I(A)$, i. e., a minimal injective C^*-algebra containing A as a C^*-subalgebra. In this paper we give a monotone complete version of the above J. D. M. Wright's result by embedding A in its injective envelope $I(A)$ (Theorem 3.1). Namely it is shown that the monotone closure $\bar{A}$ of A in $I(A)$ is a monotone complete C^*-algebra which satisfies the above properties i), ii) and iii) with $\hat{A}$ replaced by $\bar{A}$ and moreover "monotone σ-" in i) replaced by "monotone". We call $\bar{A}$ the *regular monotone completion* of A. To see that $\bar{A}$ satisfies ii) we consider the family of all unital C^*-algebras which contain A as a C^*-subalgebra and satisfy ii) (called "regular extensions" of A) and we show that, instead of $\bar{A}$, a maximal regular extension of A, written $\tilde{A}$, is realized as a monotone closed C^*-subalgebra of $I(A)$, hence that $\bar{A}\subset\tilde{A}$ satisfies ii). By the construction we have the canonical inclusions $A\subset\hat{A}\subset\bar{A}\subset\tilde{A}\subset I(A)$; however it remains open