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On presentations of the fundamental group of the 3-sphere associated with Heegaard diagrams

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1. Introduction.

We argue a property on presentations of the fundamental group of the 3-sphere S^3 associated with Heegaard diagrams. Few years ago, Prof. T. Homma and M. Ochiai made many examples of homology 3-spheres of Heegaard genus 2 by an electronic computer and picked up simply connected ones among them. In the process, they found an interesting fact that in case of 3-sphere, one of two relators of the presentation is included completely in the other one as a subword. Using this property, they showed splendidly the triviality of the group. In this paper, we show that this fact is true in certain sense for the general case of Heegaard genus n. Our proof is based on Suzuki's result [4].

2. Statement of a result.

To state our result precisely, we need some definitions about presentations of groups. Let $\langle a_1, \cdots, a_n; r_1, \cdots, r_m\rangle$ denote a presentation of a finitely generated group with generators $a_1, \cdots, a_n$ and relators $r_1, \cdots, r_m$. It will be noticed that the relator r_i is a word in the alphabet $a_1, \cdots, a_n$.

DEFINITION 1. (*Simple transformation*): We call the following transformations of relators of a presentation *simple transformations*, cf. [2]. Let b_q ($q=0, 1, \cdots, k$) denote a letter in the alphabet $a_1, \cdots, a_n$, that is, an element of the set $\{a_1, \cdots, a_n, a_1^{-1}, \cdots, a_n^{-1}\}$.

S_1) (*Cyclic reduction*); If a relator r_i is of the form $(b_1 \cdots b_j)b_0b_0^{-1}(b_{j+1} \cdots b_k)$ or $b_0(b_1 \cdots b_k)b_0^{-1}$, we say that the relator $r_i'=b_1 \cdots b_k$ is obtained from r_i by a cyclic reduction. Replace r_i by r_i'. (cf. Note 1.)

S_2) (*Cyclic permutation*); Replace a relator $r_i=b_1 \cdots b_k$ by the cyclically permuted one $r_i'=b_2 \cdots b_k b_1$.

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