J. Math. Soc. Japan
Vol. 33, No. 1, 1981

Isometry of Kaehlerian manifolds to complex projective spaces

By Kentaro YANO and Hitosi HIRAMATU

(Received March 24, 1979)

§1. Introduction.

Let M be a complex n-dimensional connected Kaehlerian manifold covered by a system of real coordinate neighborhoods $\{U; x^h\}$, where, here and in the sequel, the indices $h, i, j, k, \cdots$ run over the range $\{1, 2, \cdots, 2n\}$ and let g_{ji}, $F_i{}^h$, $\{_j{}^h{}_i\}$, ∇_i, $K_{kji}{}^h$, K_{ji} and K be respectively the Hermitian metric tensor, the complex structure tensor, the Christoffel symbols formed with g_{ji}, the operator of covariant differentiation with respect to $\{_j{}^h{}_i\}$, the curvature tensor, the Ricci tensor and the scalar curvature of M.

A vector field v^h is called a *holomorphically projective* (or *H-projective*, for brevity) vector field [**2**, **3**, **5**, **7**] if it satisfies

$$L_v\{_j{}^h{}_i\} = \nabla_j\nabla_i v^h + v^k K_{kji}{}^h = \delta_j^h \rho_i + \delta_i^h \rho_j - \rho_t F_j{}^t F_i{}^h - \rho_t F_i{}^t F_j{}^h \tag{1.1}$$

for a certain covariant vector field ρ_i on M, called the associated covariant vector field of v^h, where L_v denotes the operator of Lie derivation with respect to v^h. In particular, if ρ_i is zero vector field then v^h is called an affine vector field. When we refer in the sequel to an H-projective vector field v^h, we always mean by ρ_i the associated covariant vector field appearing in (1.1).

Recently, the present authors [**9**, **10**] and one of the present authors [**1**] proved a series of integral inequalities in a compact Kaehlerian manifold with constant scalar curvature admitting an H-projective vector field and then obtained necessary and sufficient conditions for such a Kaehlerian manifold to be isometric to a complex projective space with Fubini-Study metric.

The purpose of the present paper is to continue the joint work [**9**, **10**] of the present authors and to prove the following theorem.

THEOREM A. *If a complex $n>1$ dimensional, compact, connected and simply connected Kaehlerian manifold M with constant scalar curvature K admits a non-affine H-projective vector field v^h, then M is isometric to a complex projective space CP^n with Fubini-Study metric and of constant holomorphic sectional cur-*