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Coincidence Lefschetz numbers for a pair of fibre preserving maps

(Dedicated to Professor T. Kudo on his 60th birhday)

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Introduction.

Dold studied in [**5**, **6**] the fixed point index in connection with fibre preserving maps. In this paper we shall consider the coincidence Lefschetz numbers for a pair of fibre preserving maps, and prove various theorems which are variants of the Dold's results. Some of the results in [**5**] are obtained for generalized cohomology, but we shall be concerned with only the classical cohomology. Our method is different from that of Dold, and is the one employed by Becker and Gottlieb in their study [**1**, **9**] of the transfer homomorphism.

Let $p: E \to B$ be a fibre bundle such that each fibre $M_b = p^{-1}(b)$ $(b \in B)$ is an oriented compact m-manifold, and such that the local system $\{H^m(M_b)\}_{b\in B}$ is trivial. For simplicity, such a fibre bundle will be called an *m-orientable fibre bundle*. In this paper we shall consider frequently a pair of fibre preserving maps $f, g: E \to E'$ from an m-orientable fibre bundle $p: E \to B$ to an m-orientable fibre bundle $p': E' \to B'$. Let $h, l: B \to B'$ denote the maps induced from f, g respectively. If $h = l$ we define an element $\wedge_{f,g} \in H^m(E)$, called the Lefschetz coincidence class for f and g (see § 2). This class is fundamental in our study.

For an m-orientable fibre bundle, the integration along the fibre can be defined, and also an orientation class is defined. We review briefly these facts in § 1. If b is a coincidence point of h and l, we have the coincidence Lefschetz number $\lambda(f_b, g_b)$, where $f_b, g_b: M_b \to M'_{b'}$ $(b' = h(b) = l(b))$ are induced by f, g respectively. We study in § 2 conditions under which $\lambda(f_b, g_b)$ is independent of the choice of b, and in § 3 relations among the coincidence Lefschetz numbers $\lambda(f, g)$, $\lambda(h, l)$ and $\lambda(f_b, g_b)$ in the case when B and B' are oriented compact n-manifolds. In § 4 we deal with $\lambda(f, g)$ for equivariant maps $f, g: M \to M$, where M is an oriented connected compact manifold on which a finite group acts. We show in § 5 that the coincidence transfer homomorphism $\tau_{f,g}$ can be

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