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Immersions of Lie groups

Dedicated to the late Hsien-Chung Wang

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§1. Introduction.

Let G and L be topological groups. A group-homomorphism $f: G\rightarrow L$ is said to be an *immersion* if f is one-one and continuous. When the image $f(G)$ is dense in L the immersion f is called *dense*, and if f is a homeomorphism to $f(G)$ we shall call f an *imbedding*.

In this paper we are mainly interested in the case when G is an analytic group (=connected Lie group). First suppose that L is also a Lie group. Immersions of this kind have been studied extensively since Yosida [**19**], 1937, in which he proved that any (finite-dimensional) irreducible faithful representation of an analytic group is an imbedding. In particular, A. Malcev in [**14**], 1945, proved the following theorem.

THEOREM A. *Let G and L be analytic groups, and let $f: G\rightarrow L$ be a dense immersion. Then there exists a one-parameter subgroup (=analytic subgroup of dimension one) A of G such that*

$$L=\overline{f(A)}f(G).$$

Theorem A was also obtained in Goto [**4**], and related subjects to this theorem have been discussed in Hochschild [**11**], Djoković [**2**] and others.

Next in [**17**], 1951, van Est defined an analytic group G to be a *(CA)-group* if the group $\mathrm{Ad}(G)$ of all inner automorphisms of G is closed in the group $Aut(G)$ composed of all bicontinuous automorphisms of G, and proved the following theorem among other things:

THEOREM B. *Let G be a (CA)-group with center Z, and let L be a Lie group. If $f: G\rightarrow L$ is an immersion, then*

(i) $\overline{f(G)}=\overline{f(Z)}f(G)$.

(ii) *If $f|Z$ is an imbedding, then f is an imbedding.*

It is easy to see that (i) implies (ii), which extends some results in Yosida [**20**] and Goto [**4**]. Immersions into a more general topological group have been studied by Goto, Gleason-Palais, Lee-Wu, Ōmori, Zerling and so on. In particular, Ōmori in [**16**], 1966, generalized some part of Theorem B, and the