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On conformal diffeomorphisms between complete product Riemannian manifolds

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Introduction.

Several authors have been concerned with a problem:

Does there globally exist a non-homothetic conformal diffeomorphism between complete product Riemannian manifolds of dimension $n\geqq 3$?

Since there are conformally flat product Riemannian manifolds in the local, cf. M. Kurita [1], it is assured that a conformal diffeomorphism locally exists between two of such manifolds. On the other hand, N. Tanaka [4], T. Nagano [2], Y. Tashiro and K. Miyashita [8] showed the non-existence of such global diffeomorphism between complete Riemannian manifolds with parallel Ricci tensor, and the non-existence of infinitesimal conformal transformation generating a global 1-parameter group in a product Riemannian manifold was shown by S. Tachibana [3] in the case of compact manifold and by Y. Tashiro and K. Miyashita [7] in the case of complete manifold.

Let M and M^* be product Riemannian manifolds of dimension $n\geqq 3$, and denote the structures by (M, g, F) and (M^*, g^*, G) respectively. Under a diffeomorphism f of M to M^*, the image of a quantity on M^* to M by the induced map f^* of f will be denoted by the same letter as the original one. For example, we write g^* for f^*g^* and G for f^*G on M. If $FG=GF$ at a point $\mathrm{P}\in M$, then we say that the structures F and G are *commutative* at P with one another under f.

A purpose of the present paper is to establish the following

THEOREM. *There is no global conformal diffeomorphism between complete product Riemannian manifolds M and M^* such that the product structures F and G are not commutative under it in a dense subset of M.*

Another purpose is to give an affirmative example of a global conformal diffeomorphism making the product structures commutative.

By virtue of the well known de Rham decomposition theorem, the productness of manifolds in the theorem can be replaced by *reducibility* of manifolds by considering the universal covering spaces of them.

After preliminaries on product structures and conformal diffeomorphisms in

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