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Strong Picard principle

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Consider the punctured unit disk $\Omega: 0<|z|<1$. We view Ω as the interior of the bordered Riemann surface $\bar{\Omega}: 0<|z|\leqq 1$ with the relative boundary $\partial\Omega$: $|z|=1$ and the ideal boundary $z=0$. By a *density* $P(z)$ on Ω we mean a nonnegative locally Hölder continuous function $P(z)$ on $\bar{\Omega}$. We say that the *strong Picard principle* is valid for a density P on Ω at $z=0$ if

(1) $$u(z)=\mathcal{O}(-\log|z|) \qquad (z\rightarrow 0)$$

for every nonnegative solution u of the equation $Lu\equiv(\Delta-P)u=0$ on Ω. The *purpose* of this paper is to characterize completely those densities P for which the strong Picard principle is valid as follows:

THEOREM. *The strong Picard principle is valid for a density $P(z)$ on Ω at $z=0$ if and only if the condition*

(2) $$-\int_{\Omega-E} P(z)\log|z|\,dxdy<+\infty \qquad (z=x+iy)$$

is satisfied for a closed subset E of Ω thin at $z=0$.

The proof will be given in nos. 3-4 after some preparations in nos. 1-2. An open question related to a generalization of the above theorem to a Riemann surface is stated in no. 5.

1. We denote by $\mathcal{P}$ the family of nonnegative solutions of the equation $Lu=(\Delta-P)u=0$ on Ω with vanishing boundary values on $\partial\Omega$. The family $\mathcal{P}$ forms a halfmodule and its dimension is referred to as the *elliptic dimension* of P (or L) at $z=0$, dim P in notation. More precisely dim P is the cardinal number of the set β of extreme points of the convex set

$$\left\{u\in\mathcal{P}\,;\ -\int_0^{2\pi}\left[\frac{\partial}{\partial r}u(re^{i\theta})\right]_{r=1}d\theta=1\right\}.$$

By the Martin-Choquest theorem there exists a unique measure μ_u on β for each u in $\mathcal{P}$ such that $u=\int_\beta v\,d\mu_u(v)$. It is easy to see that dim $P\geqq 1$. After Bouligand we say that the (ordinary) *Picard principle* is valid for P on Ω at

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