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Submanifolds of a Euclidean space with homothetic Gauss map

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§1. Introduction.

In another paper of the present author some properties of the Gauss map of a submanifold in a Euclidean space were studied [8]. In that paper following topics were treated: (i) Gauss-critical submanifolds, (ii) Submanifolds such that the sectional curvature of the Grassmann manifold in each tangent plane of the Gauss image totally vanishes.

Let M be a compact orientable C^{∞} manifold of dimension m and $i: M \rightarrow E^{n}$ be an immersion such that the Gauss map $\Gamma: iM \rightarrow G(m, n-m)$ is regular. When the Grassmann manifold $G(m, n-m)$ is endowed with the standard Riemannian metric $\tilde{g}$ [7], we can consider the volume of the Gauss image $\Gamma_i(M)=\Gamma(iM)$ and letting i move in a certain domain we can define the Gauss-critical immersion. If i is a Gauss-critical immersion, $iM=M'$ is called a Gauss-critical submanifold.

Let g_i be the Riemannian metric induced from the standard Riemannian metric of E^n on the m-dimensional submanifold $M'=iM$ and G_i be the Riemannian metric induced from the standard Riemannian metric $\tilde{g}$ of $G(m, n-m)$ on the Gauss image $\Gamma_i(M)=\Gamma(iM)$. If G_i is homothetic to g_i by Γ, namely, $\Gamma: (iM, g_i) \rightarrow (\Gamma(iM), G_i)$ is a homothetic mapping, then the Gauss map is said to be homothetic.

The purpose of the present paper is to study some properties of a submanifold in E^n or of an immersion into E^n such that the Gauss map is homothetic and satisfies some other conditions.

Some years ago D. Ferus [2], [3], [4] studied immersions with parallel second fundamental form and J. Vilms [13] studied the relation between totally geodesic Gauss maps and the second fundamental form of immersed manifolds. These results overlap some part of our present study when the Gauss image is assumed to be totally geodesic. But it is to be noticed that, if the Gauss map is not assumed to be homothetic, there exist submanifolds of Euclidean space with totally geodesic Gauss image but without parallel second fundamental form [2].