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Orbits on affine symmetric spaces under the action of the isotropy subgroups

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Introduction.

Let G be a connected Lie group, σ an involutive automorphism of G and H a closed subgroup of G satisfying $(G_\sigma)_0 \subset H \subset G_\sigma$ where $G_\sigma=\{x\in G \mid \sigma(x)=x\}$ and $(G_\sigma)_0$ is the identity component of G_σ. Then the triple (G, H, σ) is called an affine symmetric space ([7, p. 223 and p. 225]). Suppose that G is real semi-simple, and consider the double coset decomposition $H\backslash G/H$.

In the case of a Riemannian symmetric space (G, K, θ) of noncompact type the double coset decomposition is the Cartan decomposition $G=KA_{\mathfrak{p}}K$. Secondly consider an affine symmetric space $(G\times G, \Delta G, \sigma)$ where G is real semi-simple, ΔG denotes the diagonal of $G\times G$, and σ is the mapping $(x, y)\rightarrow(y, x)$ $(x, y\in G)$. If we identify $G\times G/\Delta G$ with G in the natural way, then the structure of $\Delta G\backslash G\times G/\Delta G$ is, for the most part, known by the following Harish-Chandra's theorem (see [3, p. 102], [4, p. 556] and [12, p. 113]).

THEOREM. *Let G' be the set of regular elements in G, $\{\mathfrak{j}_i \mid i=1, \cdots, r\}$ representatives of conjugacy classes of Cartan subalgebras in $\mathfrak{g}$, and J_i the Cartan subgroup associated with $\mathfrak{j}_i$. Then*

$$G'=\bigcup_{i=1}^{r}\bigcup_{x\in G} xJ_i'x^{-1}$$

where $J_i'=J_i\cap G'$.

In this paper we will extend this theorem to an arbitrary affine symmetric space (G, H, σ) such that G is real semisimple.

Let φ be the mapping of G into G defined by $\varphi(g)=g\sigma(g)^{-1}$ for $g\in G$ (see [1], [8, p. 182]). Then G/G_σ and $\varphi(G)$ are diffeomorphic by this mapping, and the H-orbits on G/G_σ correspond to the H-orbits on $\varphi(G)$ under the action $(h, x)\rightarrow hxh^{-1}$ $(h\in H, x\in\varphi(G))$. Let $\mathfrak{g}$ and $\mathfrak{h}$ denote the Lie algebras of G and H, respectively, and let the automorphism σ of $\mathfrak{g}$ be the one induced by the automorphism σ of G. Put $\mathfrak{q}=\{X\in\mathfrak{g} \mid \sigma(X)=-X\}$. A subspace $\mathfrak{a}_{\mathfrak{q}}$ of $\mathfrak{q}$ is called an A-subspace if the following two conditions are satisfied: (i) $\mathfrak{a}_{\mathfrak{q}}$ is