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The first Chern class and holomorphic symmetric tensor fields

To Fumio Sakai and Carmen Silva

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§ 1. Introduction.

Let M be a complex manifold and TM (resp. T^*M) be the holomorphic tangent (resp. cotangent) bundle of M. Let S^mTM (resp. S^mT^*M) be the m-th symmetric tensor power of TM (resp. T^*M). Thus $\Gamma(S^mTM)$ (resp. $\Gamma(S^mT^*M)$) denotes the space of holomorphic contravariant (resp. covariant) symmetric tensor fields of degree m.

The purpose of this paper is to answer affirmatively the following question raised by F. Sakai[1]. If M is a Kähler K3 surface, are $\Gamma(S^mTM)$ and $\Gamma(S^mT^*M)$ trivial? (For Kummer surfaces and quartic surfaces, the answer has been known to Sakai). We shall show that if M is a compact, simply connected Kähler manifold with vanishing first Chern class, then

$$\Gamma(S^mTM)=\Gamma(S^mT^*M)=0 \qquad \text{for} \quad m>0\,.$$

Several related results, some applicable to manifolds of general type, will be proved on the way. The main ingredients of the proof consist of (i) Bochner's method of proving vanishing theorems, (ii) Berger's classification of irreducible holonomy groups, (iii) representations of $SU(n)$ and $Sp(n)$, and (iv) Yau's solution of the Calabi conjecture. Using (i), (ii) and (iii) we obtain differential geometric results in which assumptions are stated in terms of the Ricci tensor. Applying (iv) we can restate the results in terms of the first Chern class.

§ 2. Ricci tensor and holomorphic symmetric tensor fields.

By the well known method of Bochner we prove the following

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1) The question was raised and answered aboard Motorship "Carmen Silva" on the Rhine during the 19th "Arbeitstagung", 1978.