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Holomorphic continuation of solutions of partial differential equations across the multiple characteristic surface

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§ 1. Introduction.

Let $P(z, \partial_z)$ be a linear partial differential operator of order m with holomorphic coefficients defined near a point p in C^n and Ω be an open set such that the point p belongs to the C^2-boundary $\partial\Omega$. In this paper we shall study the following problem:

(Q) Let u be a holomorphic solution of $Pu=f$ in Ω, where f is holomorphic near p. What conditions on Ω and P garantees that u can be continued across p?

The results already obtained to this problem are in the cases where $\partial\Omega$ is tangent to the holomorphic hypersurface S which is non-characteristic with respect to P (Bony-Schapira [2], Zerner [10]) or simple characteristic (Pallu de La Barriere [5], Persson [6], Tsuno [7]) or some other special cases ([5], [8], [9]). In [5], the case where the tangential operator of P on $\partial\Omega$ has the regular characteristic variety is studied. In [6] Persson also studied the general case using the so called "cones of analytic continuation". The purpose of this paper is to extend these results to the case where $P(z, \partial_z)$ is highly degenerated at p. Since the problem (Q) is invariant under the holomorphic change of variables, it is desirable to describe the results free from the choice of the local coordinates. But the treatment of the normal direction of $\partial\Omega$ at p and the tangential directions of $\partial\Omega$ at p is different, so we introduce the weighted coordinates in the next section. The weighted coordinates are systematically used to determine the type of $\partial\Omega$ at p by T. Bloom and I. Graham [1]. The weighted coordinate system used in this article is the simplest one such that the complex normal coordinate z_1 of $\partial\Omega$ at p is assigned the weight 2, while the complex tangential coordinates $z_2, z_3, \cdots, z_n$ are each assigned the weight 1. In the second section, some invariances are shown under the equivalent change of the weighted coordinates. Then in the third section, we state the basic theorem under some fixed local coordinates. The