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Hölder estimates on higher derivatives of the solution for $\bar{\partial}$-equation with C^k-data in strongly pseudoconvex domain

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§ 0. Introduction.

In 1971 Kerzman [4] showed there exists a solution of $\bar{\partial}$-equation with bounded data which is Hölder continuous for any exponent smaller than 1/2. Since then many results have been obtained concerning this problem. Henkin-Romanov [3] and Range-Siu [5] proved the exact 1/2-Hölder estimate. Moreover Siu [6] showed the Hölder continuity of higher derivatives of the solution assuming the data are sufficiently smooth. In this paper we shall improve Siu's result and get a new estimate which is sharper in some tangential directions. We follow the method of Siu [6]; however, various parts of his calculus are ameliorated. I thank Professor H. Tanabe, who encouraged me to write this paper and corrected my manuscript.

0.1. Notations.

Let Ω be a bounded strongly pseudoconvex domain in $\boldsymbol{C}^n$ with C^N-boundary. We assume that Ω is represented as $\{z\in\boldsymbol{C}^n;\ \rho(z)<0\}$, where ρ is a function of class C^N and in some neighborhood of $\partial\Omega$ is strictly plurisubharmonic and satisfies $d\rho\neq 0$. We use the following notations;

$$D_j=\frac{\partial}{\partial z_j}=\frac{1}{2}\Big(\frac{\partial}{\partial x_j}-i\frac{\partial}{\partial y_j}\Big),\quad \bar{D}_j=\frac{\partial}{\partial \bar{z}_j}=\frac{1}{2}\Big(\frac{\partial}{\partial x_j}+i\frac{\partial}{\partial y_j}\Big),$$

$$\|u\|_0=\sup\{|u(z)|;\ z\in\Omega\},$$

$$\|u\|_\varepsilon=\sup\{|u(z)-u(\zeta)|/|z-\zeta|^\varepsilon;\ \zeta,\ z\in\Omega,\ \zeta\neq z\}+\|u\|_0$$

$$\|u\|_k=\max\{\|D^\alpha\bar{D}^\beta u\|_0;\ |\alpha|+|\beta|\leqq k\},$$

$$\|u\|_{k+\varepsilon}=\max\{\|D^\alpha\bar{D}^\beta u\|_\varepsilon;\ |\alpha|+|\beta|\leqq k\}$$

where $k\in\boldsymbol{N}$ and $0<\varepsilon<1$. For a form $f=\sum f_i d\bar{z}_i$,

$$\|f\|_k=\max\{\|f_i\|_k;\ 1\leqq i\leqq n\}.$$