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On (x)-complexes

By Yasuji TAKEUCHI

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Let N be a finitely generated module over a noetherian local ring R with maximal ideal $\mathfrak{m}$. It is well known that a maximal R-sequence and a maximal N-sequence have connections with a minimal injective resolution of N. For example, the length of a maximal R-sequence, namely the depth of R, is equal to the length of a minimal injective resolution of N, namely the injective dimension of N, if it is finite, and the length of a maximal N-sequence, namely the depth of N, is equal to the minimal integer of i with $\mu^i(\mathfrak{m}, N)>0$ where $\mu^i(\mathfrak{m}, N)$ is the dimension of an $R/\mathfrak{m}$-vector space $\mathrm{Ext}_R^i(R/\mathfrak{m}, N)$. But we have thought there are more connections between them. In particular we are interested in studying possible connections between the terms of an R-sequence or of an N-sequence, and the terms of a minimal injective resolution of N.

First we shall introduce a complex associated to a minimal injective resolution of N for a sequence of elements in $\mathfrak{m}$ [see Definition 1]. This complex characterizes some N-sequence. We shall study properties of this complex. In particular we shall give, using a term of this complex, a necessary and sufficient condition for the following conjecture of Bass to hold: a noetherian local ring is Cohen-Macaulay if it possesses a finitely generated module of finite injective dimension. Moreover we shall show some property of a minimal injective resolution, applying this complex.

Throughout this note, R is a noetherian local ring with a unique maximal ideal $\mathfrak{m}$. The unlabeled Hom and Ext mean Hom_R and Ext_R, respectively.

We begin by introducing a definition. Let $x_0, x_1, \cdots, x_r$ be a sequence of elements in $\mathfrak{m}$. We denote this sequence by $(x_0, x_1, \cdots, x_r)$ or, for brevity, (x). The ideal generated by $x_0, x_1, \cdots, x_r$ is also denoted by $(x_0, x_1, \cdots, x_r)$ or (x). Let N be an R-module and $0 \longrightarrow N \xrightarrow{d^{-1}} E^0 \xrightarrow{d^0} E^1 \xrightarrow{d^1} \cdots$ be a minimal injective resolution of N.

DEFINITION 1. Let $N_{(x)}^0=\{e\in E^0 \mid x_0 e\in d^{-1}(N)\}$. For any integer i with $0<i\leqq r$, we define inductively $N_{(x)}^i$ as follows; $N_{(x)}^i=\{e\in(0:(x_0, x_1, \cdots, x_{i-1}))_{E^i} \mid x_i e\in d^{i-1}(N_{(x)}^{i-1})\}$. For $i>r$, $N_{(x)}^i=(0:(x_0, x_1, \cdots, x_r))_{E^i}$. Each $N_{(x)}^i$ is a submodule of E^i. Each d^i induces an R-homomorphism: $N_{(x)}^i\to N_{(x)}^{i+1}$, denoted again by d^i. In this case we have a complex of R-modules