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A class of infinitesimal generators of one-dimensional Markov processes

II. Invariant measures

By H. LANGER and W. SCHENK

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It was shown in [4] that an operator of the form (1) below with boundary conditions of Feller-Wentzell type is the infinitesimal generator of a strongly continuous nonnegative contraction (s. c. n. c.) semigroup $(T_t)_{t\geq 0}$ in $\boldsymbol{C}=C([0, 1])$*) or a subspace of $\boldsymbol{C}$. In this note we continue the study of these operators. The main result is that the semigroup $(T_t^*)_{t\geq 0}$ or the corresponding Markov process have a unique invariant measure μ_0 with supp $\mu_0=[0, 1]$ if only the boundary conditions are "not too degenerated". This seems to be rather evident as the operator (1) contains a diffusion term $D_m D_x$. However the analytical proof of this fact we could give (Theorem 5) is not so short. Further it is shown that μ_0 is in (0, 1) absolutely continuous with respect to the measure m.

In a following note we shall continue the study of this class of Markov processes along the lines of [6]. In particular, we shall investigate the limit behavior of the transition probabilities if $t\rightarrow\infty$ and derive Kolmogorov's equations for the densities of the transition probabilities (with respect to μ_0). As an important tool, the extension of the semigroup $(T_t)_{t\geq 0}$ to $L^2(\mu_0)$ (with scalar product denoted by $[\cdot, \cdot]$) is considered. The explicit expressions of $[Af, f]$ and its real and imaginary parts, given at the end of this paper, will play an essential role in this investigation.

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1. Preliminaries.

Let m, b and the family of measures n_x, $x\in[0, 1]$, have the same properties as in [4], [5] that is m is a strongly increasing continuous function

*) In [4] only real spaces have been considered, here, however, $\boldsymbol{C}$ is supposed to be complex. It is easy to see ([5], p. 106), that the statements quoted above are true for the corresponding complex spaces.