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Free group actions of $Z_{p,q}\times Z_h$ on homotopy spheres

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Introduction.

Let $Z_{p,q}$ be the metacyclic group with presentation

$$\{x,\ y\,|\,x^{p}=y^{q}=1,\ yxy^{-1}=x^{\sigma}\}\ ,$$

where p is an odd integer, q an odd prime, $(\sigma-1,\ p)=1$, and σ is a primitive q^{th} root of 1 mod p. Denote by Θ_{n} the group of homotopy spheres, and by $\Theta_{n}(\partial\pi)$ the group of homotopy spheres which bound parallelizable manifolds. Then Petrie [5] proved that for each $\Sigma\in\Theta_{2q-1}(\partial\pi)$ there is a free smooth action of $Z_{p,q}$ on Σ. This theorem will be generalized as follows in this paper.

THEOREM. *Let Z_{h} denote a cyclic group of order h and assume $h=2^{n}h'$, $(2,\ h')=1$. If n takes 0, 1, or 2 and $(h',\ pq)=1$, then for each $\Sigma\in\Theta_{2q-1}(\partial\pi)$ there is a free smooth action $Z_{p,q}\times Z_{h}$ on Σ.*

Our theorem follows immediately from the following two propositions.

PROPOSITION 5.7. *There exists a free smooth action of $Z_{p,q}\times Z_{h}$ on some homotopy sphere $\Sigma\in\Theta_{2q-1}(\partial\pi)$. Here $(h,\ pq)=1$.*

PROPOSITION 6.1. *Let m be any integer $\geqq 1$. Assume $h=2^{n}h'$ where $n=0$, 1, or 2 and $(h',\ pq)=1$. If $\Sigma\in\Theta_{4m+1}$ admits a free $Z_{p,q}\times Z_{h}$-action, then $\Sigma\,\#\,\Sigma_{0}$ admits a free $Z_{p,q}\times Z_{h}$-action, where Σ_{0} generates $\Theta_{4m+1}(\partial\pi)$.*

Our methods are analogous to those in Petrie [5]. §§ 1-4 are preliminaries for Proposition 5.7 which is proved in § 5. In § 6, we prove Proposition 6.1 by applying a theorem of Browder [1].

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1. Construction of a $Z_{p,q}\times Z_{h}$-action.

We set $\pi=Z_{p,q}\times Z_{h}$ for the groups $Z_{p,q}$ and Z_{h} in Introduction, where $(h,\ pq)=1$. We denote by π_{p}, π_{q} the cyclic subgroups generated by x, y respectively. Let Z_{ph} be a cyclic group of order ph. Since $(p,\ h)=1$, there exist integers m and n such that $mp+nh=1$, and an isomorphism of Z_{ph} to